Gaines, B. R. (2010). Human rationality challenges universal logic. **Logica Universalis** 4, 163-205.

Human Rationality Challenges Universal Logic

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Abstract

Tarski's conceptual analysis of the notion of *logical consequence* is one of the pinnacles of the process of defining the metamathematical foundations of mathematics in the tradition of his predecessors Euclid, Frege, Russell and Hilbert, and his contemporaries Carnap, Gödel, Gentzen and Turing. However, he also notes that in defining the concept of *consequence* "efforts were made to adhere to the common usage of the language of every day life." This paper addresses the issue of what relationship Tarski's analysis, and Béziau's further generalization of it in *universal logic,* have to reasoning in the everyday lives of ordinary people from the cognitive processes of children through to those of specialists in the empirical and deductive sciences. It surveys a selection of relevant research in a range of disciplines providing theoretical and empirical studies of human reasoning, discusses the value of adopting a universal logic perspective, answers the questions posed in the call for this special issue, and suggests some specific research challenges.

1 Introduction

The call for papers for this special issue raises a number of deep issues that range far more widely than is normal for this journal, or any other journal focused on technical aspects of logic:-

1. Do all human beings have the same capacity of reasoning?
2. Does reasoning evolve?
3. Do we reason in different ways depending on the situation?
4. Do the different systems of logic reflect the diversity of reasonings?
5. Is there any absolute true way of reasoning?

These are questions related to the cultural and psychological foundations of reasoning and the kernel of logical principles that seem to be implicit in those foundations. The issues raised are normally addressed in many different disciplines such as psychology, anthropology, sociology economics and linguistics. Universal logic, with its origins in the principles of formal logics developed to provide foundations for mathematics as an autonomous, abstract system independent of human reasoning, seems very remote from naturalistic, empirical, human studies. The call for papers may be seen as an invitation and a challenge to explore the relevance and value of a universal logic perspective on aspects of human rationality that are less prescriptive than our highly specialist techniques for formal reasoning but might be modeled as precursors to them.

What value might a *universal logic* perspective (Béziau, 2006; 2007) provide to theoretical and empirical studies of human reasoning? At an abstract level it focuses on consequence operators as the primary unit of analysis in modeling reasoning, and represents them as relations` on the powerset generated by an arbitrary set, with no *a priori* constraints on what constitutes the elements of the set, algebraic constraints on admissible and equivalent sets, or axioms constraining them. This extreme generality enables constructs and insights from many theoretical developments and applications of deductive reasoning to be subsumed and integrated within unified frameworks that encompass proof theory, model theory, notions of structurality and compositionality, definition of the logical constants from structural constraints on the consequence operator, and so on.

At a conceptual level, a *universal logic* perspective deconstructs the notion of logical reasoning, directing attention to each assumption made in any logical system, the implications of making it, the interactions between various assumptions, and so on. It makes it possible to question whether those assumptions are appropriate or realized in different situations, and to vary the assumptions according to the situation if it is appropriate to the theory, model or empirical study.

At an applied level, questions raised within the human reasoning literature across many disciplines might be clarified within a universal logic framework. Stenning and Lambalgen (2008) have explained many of the puzzling results from empirical studies of human reasoning as due to variant interpretations that entail the use of different logical systems. Cellucci (2008) has discussed the parallels between the basic anticipatory processes of all organisms and the development of formal proofs in mathematics and noted that “analytic proof is based on the procedure by which organisms provide for their most basic needs.” Van Benthem (2006) has argued that the scope of logic should extend beyond proof and include “mistakes and revisions, questions and debates, and much, much more.”

These and similar developments involve three basic paradigm shifts: first, to extend the scope of the notion of logical inference to include any process that can be modeled by the abstract notion of consequence; second, to see particular logical principles as situation-specific, and hence dynamic; and, third to extend what is being modeled as a logical process to encompass the entire situation under study including if appropriate, the problem situation, its design, and the process of drawing inferences about the outcomes.

The call for papers may be seen as an invitation to discuss such issues, and this article dos so firstly in terms of how they are addressed from the perspectives of other disciplines that have studied them, secondly how they might be addressed from a universal logic perspective, and thirdly how these two perspectives complement and enrich one another. It concludes by collecting some interesting research issues that arise from these analyses and presenting them as ‘challenges’ to the universal logic research community.

2 What is rationality?

Central to the questions posed and all the literatures concerned with the study of human reasoning are the methodological issues of what is meant by rationality. What is the phenomenon that is to be studied, how will it be recognized, modeled and assessed? A common issue across all disciplines is that it becomes recognized that there is more than one notion of rationality and that methodological problems are arising because different scholars are using different criteria.

Shore (1996, p.169) notes that “many of the following types of rationality have been evoked in the rationality debate, but often with no acknowledgement of their unstated assumptions or even that rationality was subject to multiple definitions:”

1. *Logical rationality* assumes that beliefs follow canons of formal logic*;*
2. *Contemplative reason* assumes that beliefs or acts are based on clearly thought out principles rather than on emotion or desire;
3. *Conscious reason* assumes that actions are subject to conscious awareness;

4. *Causal reason* assumes that actions or statements can be understood as being coherently caused or motivated;
5. *Calculating rationality* assumes that means are “realistically” adjusted to one’s goals or interests;
6. *Functional rationality* assumes that a set of beliefs or acts is adaptive for an individual or group;
7. *Communicative rationality* assumes that behavior is adjusted to demands of social communication.
8. *Empirical rationality* assumes that statements or behaviors are consistent with an accurate perception of reality;
9. *Contextual rationality* assumes that acts or statements are “logical” in terms of an often hidden context of supporting beliefs or acts with which they are functionally integrated;

The notion of *logical rationality* might seem the appropriate one for universal logic purposes, and there is a substantial literature on empirical studies of whether human reasoning can be modeled in terms of formal logic. However, it should be noted that all of the other definitions also have logical connotations and may be regarded as providing different perspectives on rationality rather than conflicting definitions.

The opposition between reason and emotion in the definition of *contemplative reason* might seem to place emotion outside logic, but the literature on emotion suggests otherwise. The term originates from Aristotle’s *Nicomachean ethics* as the highest virtue but he does not contrast it with emotion and provides a thoroughly rational model of the basis of emotion in his *Rhetoric*. This accords with modern theories that treat emotion as rational behavior (Brun, Doguoglu and Kuenzle, 2008; Gratch, Marsella and Petta, 2009), a cognitive response to a failure of anticipation (Kelly, 1955; McCoy, 1981) indicating that the logical model of a situation needs to be revised. This suggests links with the AGM logic of theory change (Alchourron, Gardenfors and Makinson, 1985), with positive emotions indicating a preference that unanticipated consequences be preserved, and negative that they be removed.

The notion of *conscious reason* may be interpreted in terms of the reasoning process itself being itself explicitly modeled by the reasoning agent. Moshman (1990) has termed such modeling *metalogical* and traced its development through different phases of life. Metalogical inferences include the agent being able to examine the basis for its conclusions in terms of the inferences made, assignment of truth values, necessity of conclusions, implicit and default assumptions, and so on. Such a review might arise if the conclusions are contradictory, or otherwise unacceptable, and lead to revisions of truth assignments, defaults used, and so on. This may be modeled in terms of the logics of reflection, explanation and theory change, but capturing all the phenomena of human explanation is problematic (Keil, 2006).

The definitions of *causal*, *calculating* and *functional rationality* all involve hypotheses that the reasoning serves some explicit or implicit purpose, and these are problematic in the literature because it may always be possible to construe apparently irrational behavior as serving some rational purpose (Cohen, 1981).

The notion of *communicative rationality* in the fourth definition may be interpreted as *conscious reason* being communicable to others, raising issues of *mutual knowledge* (Smith, 1982), *conversational implicature* (Grice, 1989), and so on, as well as those of justification in a proof-theoretical sense (Cellucci, 2008). Willard (1989) has analyzed communicative rationality in depth in his *Theory of Argumentation* and notes that "one's 'rationality' in a community is one's sanity, competence and predictability—one's authenticity as a native, one's bona fides as a group member" (p.265). This socially-grounded definition supervenes on the other definitions which can be viewed, from this perspective, as illustrating the types of criteria that a community might apply.

The definitions of *empirical rationality* and *contextual rationality* have obvious interpretations in notions of *correspondence* and *coherence* truth, with connotations of both the well-documented philosophical issues that these notions entail and also the range of theories that have been developed to model them formally in logical terms. In addition, hypotheses of empirical and contextual rationality are often used to infer agents' models of their situations and their implicit belief system, and this inference process is itself an interesting target for logical modeling.

3 Issues of empirical studies of human rationality

The questions posed for this special issue involve assessing human rationality from empirical data and raise methodological issues of inferring the internal processes of a system from its behavior. One problem that arises is that such inference is impossible without presuppositions that can radically affect the outcome. For example, if it is assumed that the structure of a stochastic system is deterministic the resulting models grow in size indefinitely and have no predictive power (Gaines, 1976). This may be seen as an example of empirical research in general being essentially *theory-laden* (Kordig, 1971), but the issue is more problematic in human research because the brain is one of the most complex systems known and only accessible through channels of interaction that are of very low bandwidth relative to its complexity.

An insidious aspect of theory-ladeness when modeling a system as plastic and adaptive as the brain is the way in which the researcher's choice of the family of tasks that provides the subjects' environment for the study becomes reflected in the inferred model of the subjects. Craik (1943) hypothesized that the simplest explanation of human skill was that the person formed a *model* of the task, and Conant and Ashby (1970) later showed in very general terms that "every good regulator of a system must be a model of that system." However, this phenomenon can be the source of artifacts arising from the choice of tasks where the outcome is more a model of the task than the agent solving it. For example, researchers on perceptual-motor skills who modeled human control behavior when controlling linear dynamic systems with linear "describing functions" populated the literature with complex models having little predictive power that later became explicable as generated by the interaction between the researchers' assumption of linearity and the actual discontinuous, "bang-bang" mechanism of human movement (Gaines, 1969).

Attempted resolutions of the general theory-laden problem commonly appeal to cognitive models of the researcher and research community (Estany, 2001), which emphasizes that the researchers exhibit the same phenomena of human rationality as the subjects whose rationality they are studying. Conversely, the subjects act as 'personal scientists' (Kelly, 1955; Shaw, 1980) in exploring the situations in which they are involved and modeling them within the frameworks of their own previous experience and preconceptions. Another major source of artifacts in human studies is that the subjects' models of their situations may be very different from those intended by the experimenter, and may include reasoning based on their models of the experiment and experimenter as much as those of the task (Wetherick, 1995). These considerations suggest that any use of empirical studies to develop or evaluate a logical model of human rationality needs to include the researchers and their preconceptions as data on a par with the outcomes of the studies and to take into account the interaction between the researchers' preconceptions and those of their subjects.

Blomberg's transcription of Kant's logic lectures from the early 1770s show Kant's appreciation of impact of presuppositions on empirical studies, summed up by his statement, "The logical essence is far smaller than the real essence. It only comprehends what we represent in the thing." (Kant, 1992, p.91). The notion of a 'real essence' is problematic nowadays with the widespread acceptance of a Deweyian, pragmatic, instrumentalist, constructivist stance (Margolis, 2010), but the 'essence' can be construed in such terms as those features of a model that are necessary to its utility in anticipating the system being modeled.

Carnap (1950, p.7) provides conceptual analysis of scientific modeling as *precisification* from an informal explicandum to a formal explicatum, noting that:

1 The explicatum is to be similar to the explicandum in such a way that, in most cases in which the explicandum has so far been used, the explicatum can be used; however, close similarity is not required, and considerable differences are permitted.
2 The characterization of the explicatum, that is, the rules of its use (for instance, in the form of a definition), is to be given in an exact form, so as to introduce the explicatum into a well-connected system of scientific concepts.
3 The explicatum is to be a fruitful concept, that is, useful for the formulation of many universal statements (empirical laws in the case of a nonlogical concept, logical theorems in the case of a logical concept).
4 The explicatum should be as simple as possible; this means as simple as the more important requirements (1), (2), and (3) permit.

The dynamics of moving the explicatum closer to the explicandum may be modeled within a universal logic framework as one of determining whether additional statements about the explicandum are consequences of the explicatum or, if not, consistent with it, and adjusting the explicatum to remove inconsistency and derive the desired consequences. This is also Piaget's (1985) *equilibration* process of human development as *assimilation* and *accommodation* which can be modeled within a logical framework as one of evolutionary theory change (Hooker, 1995).

These methodological issues have been recognized as a source of problems in the research literatures central to the questions raised in this special issue. Piaget (1973, p.25) notes the difficulties in determining whether his own structuralist models of the development of rationality in children, "constitute simple 'models' in the service of theoreticians or whether they should be considered as inherent to the reality under study." Dennett (1987) has argued that any attempt to derive the intentionality of living systems from their behavior will fail, and we must accept that it is the modeler who finds it convenient to model certain classes of system from an *intentional stance*; that is, instrumental rationality on the part of the modeler not of the system being modeled. Some of the major contributors to research on human rationality have noted that, "critics of the application of classical rational principles typically assume that such principles explain behavior by assuming that the mind performs rational calculations. We believe that is a crucial mischaracterization of the project of rational explanation in the social and biological sciences, which aim for rational description, without ascribing rational calculations to the cognitive system." (Chater, Oaksford, Nakisa and Redington, 2003, p.66).

Acceptance that the rationality involved is that of the researchers' development of logical models, not of the ascription of reasoning processes to the agents being studied may appear to trivialize research on human rationality, and to bypass most of the specific questions posed for this special issue. The majority of researchers on human rationality are concerned with the reasoning processes of those they are studying rather than their own. However, the issue can be resolved if one views those under study as themselves being epistemological agents developing rational descriptions of their own reasoning processes (Carruthers, 2009). A simple creature may exhibit goal-seeking stimulus-responses reflexes based on particular characteristics of our corner of the universe such as causality and object identity that may be modeled logically as material conditionals without any imputation that it is 'reasoning' even though its behavior may be a rational adaption to its environment. A more complex creature may model such behavior in other creatures in the same way as it models causal behavior in its physical environment, again without any imputation of significant reasoning. One more complex still may model its own modeling activity and use, perhaps, its uncertainty as a signal to change its behavior. And so on, until eventually, there are creatures which can 'explain' their own behavior to others in linguistic terms, develop accounts of rationality in society, and so on. At some stage of this increasingly reflective process it will come to be agreed that 'reasoning,' 'thought processes,' or 'rationality' has been exhibited.

These considerations have affected experimental designs in the empirical study of animal and human reasoning. Wright (2001) has discussed how researchers in child development have critiqued their studies for methodological flaws in imputing rationality through behavioral studies, and notes that "Piaget attempted to dissociate genuine logical transitive inference from lower forms of pseudo-deductive transitive inference by requiring transitive responding along with verbal justifications." Such a requirement can be seen as *metalogical*, to not only behave in such a way as to solve a logical problem but also to be able to exhibit knowledge of the logical basis of the solution.

As reflective behavior develops it also becomes more difficult to model in terms of complex structure of goals involved. Schütz (1943) has analyzed the role of human rationality in phenomenological terms drawing upon the American pragmatists to clarify the issues. From James he takes the notion that all concepts have a *fringe* of aspects that can be lost in an attempt to define them logically. From Dewey he takes the notion that thought is provoked by the *gap* between a desired and an actual situation, and that action is guided by the *anticipation* of its consequences. He develops a rich analysis of rational behavior based on the network of knowledge that an agent might have to take into account: an end's relationship with other ends; the consequences and side-effects of achieving an end; the means appropriate to the end; the interaction of such means with other ends and means; the accessibility of those means; the construal that others might place on the actions; its interaction with their own planned actions; and so on.

Schütz's analysis shows that the logical explication of human rationality involves a far more sweeping notion of the set of consequences than a superficial analysis might suggest. It is echoed in critiques of experimental studies of human rationality that note that the subjects being studied have different perspectives on the situation than the experimenters intend, and that this may explain their apparent irrationality from the experimenter's point of view (Stenning and van Lambalgen, 2008). However, the behavior is still amenable to logical analysis if the set of possible consequences being considered can be identified. The indefinite recursion involved in such a process may be seen as the theme of Heidegger's (1962) phenomenological analysis of the lifeworld in *Being and Time*. Dreyfus (1979) has used this analysis to characterize the limits of logicist approaches to modeling human skilled behavior in artificial intelligence and cognitive science.

From one perspective the issues raised in phenomenology are negative to the possibility of a logical framework being able to contribute to the understanding of human processes. There will always be aspects of the phenomenon beyond logical analysis, and improved analysis taking into account more aspects can lead to such a detailed history of idiosyncratic behavior that it may be impossible to derive any general conclusions. However, the history of science provides major counter-examples that illustrate what can sometimes be achieved; for example, Adam Smith's (1776) demonstration of how the simplicity of an effective market economy emerges from the complex interactions of the rational self-interests of interacting agents. Computer simulation of the behavior of societies of logical agents is now being used to demonstrate how socially rational phenomena emerge (Epstein, 2006), and it would be interesting to model this from a universal logic perspective to determine whether the global rationality can be derived analytically from logical models.

Another problem in interpreting empirical studies of human reasoning is that even the simplest problems allow for many forms of solution differing widely in their structure and implementation. There is a wide variety of ways in which organisms can achieve similar goals in similar circumstances, and there may be little commonality among them other than that the goal is achieved (Rozencwajg and Corroyer, 2001). In empirical studies it is reasonable practice for particular researchers or research schools to champion one form of model and develop experimental situations in which that model successfully anticipates the outcome and other competing models do not (Kitcher, 1990; De Langhe, 2010). It is important to establish the boundaries of the phenomena accountable by each form of model. However, there is no implication that particular agents all use the same problem-solving strategy. On the contrary, it is reasonable to suppose that any strategy that is effective is likely to be employed by some agents, and that individual agents will employ a heterogeneous mix of strategies (Carruthers, 2004).

4 Development of rationality

One common thread in the questions posed for this special issue is whether reasoning differs among people at different times, eras, cultures, and so on, and do any differences correspond to different forms of logic. The empirical literature on the development of reasoning across a lifetime, across cultures, across disciplines, across species, through evolution, and so on, is vast and confusing if take in isolation. Empirical studies generally address particular debates at particular times in particular literatures, and are meaningful only to specialists situated in those debates. Surveys that summarize entire genres of such research over many decades are more useful for those seeking more general models. The coherent intensive research areas of relevance to the special issue are those of developmental psychology, largely associated with Piaget's seminal research. This has been extended to the very young and to the reasoning of other species at one extreme, and to the ongoing development of adults and specialists in the exact sciences at the other. It is this broad spectrum of studies in reasoning from the simplest stimulus-response driven behavior through to the multiple meta-levels of philosophy, metamathematics and metalogic that it is reasonable to attempt to summarize in addressing the questions posed for this special issue.

Piaget (1970, p.1) notes that his research program of "genetic epistemology attempts to explain knowledge, and in particular scientific knowledge, on the basis of its history, its sociogenesis, and especially the psychological origins of the notions and operations on which it is based." He was highly influenced by the Bourbaki school of mathematics and the cybernetics of Ashby, and analyzed individuals as cybernetic agents coming to model their worlds through their interactions with them. His partnerships with those having technical knowledge of advances in formal logic such as Beth and Garcia enabled Piaget to explore the role of modern developments in logic in the exposition of his theories (Beth and Piaget, 1966; Piaget and Garcia, 1991). His notion of the *equilibration* (Piaget, 1985) of cognitive structures as an interplay of *assimilative* use of existing models to account for new experiences and *accommodative* adjustment of those models to improve the account has clear connections to modern logical models of theory change. His theory of the *stages* of the equilibration process has not only been a core model in developmental psychology and educational theory and practice, but has also been used in anthropology to characterize primitive thought (Hallpike, 1979; Wynn, 1985) and by Piaget and others to model the history of science (Kitchener, 1987; Piaget and Garcia, 1991).

Any overview of Piaget's empirical studies of the development of reasoning in children is inevitably superficial but the notion of *stages of cognitive development* is significant to the questions for this special issue. It can be summarized in terms of rationality developing in a succession from: a *sensorimotor stage* commencing with motor reflexes at birth which the child generalizes to a wider range of situations and coordinates into increasingly lengthy chains of behavior; a *preoperational stage* in which children acquire representational skills through mental imagery and language but model their worlds egocentrically; a *concrete stage* in which children are able to take another's point of view, take into account multiple perspectives and represent transformations as well as static situations but they cannot yet use abstractions; a *formal stage* in which children are capable of thinking logically and abstractly and can also reason theoretically. Generally, the transitions between the stages occur around ages of 2 years, 6 years and 11 years, but there is wide variation in timing and breadth and depth of achievement, and the different stages may be reached in different domains at different times or not at all.

The differential rates and levels and domain dependencies of the development of formal reasoning skills are of major significance to the role of formal logic in human reasoning. Piaget notes that "the more a subject advances in development, the more a diversity is found in given tasks and in individual abilities…I see no contradiction in observing individuals of any age who function quite logically in the limited area of their specialty but are quite retarded and on an inferior level in those fields that are outside their specialty." (Voyat, 1982, quotation p.23). Moshman (1990, p.222) has focused on the development of metalogical aspects of formal reasoning and has identified four stages in the development of metalogical understanding: implicit inference about content; explicit inference based on an implicit logic; logical necessity explicitly understood on the basis of implicit metalogical awareness; explicit reflection on metalogic. He has surveyed research on cognitive development beyond childhood (Moshman, 1998) and analyzed the development of rationality, noting that "If by rationality we mean conformity to rules of logic, then even preschool children are substantially rational in their inferential processes. If by rationality we mean metacognitive awareness, knowledge, and control of inferential processes, however, then rationality develops over a period of many years that often extends well beyond childhood without ever attaining a definitive state of maturity." (Moshman, 2004).

Experimental psychologists have investigated formal reasoning in laboratory situations using artificial tasks to probe, for example, whether people can apply logical strategies based on *modus ponens* and *modus tollens* (Manktelow and Chung, 2004). The paradigmatic example with a massive derivative literature is Wason's (1972) *selection task* of which the original form was to show subjects four cards having the symbols E, K, 4 and 7, the rule that "If a card has a vowel on one side, then it has an even number of the other", and ask them to specify the cards which need to be turned over in order to determine if the rule is true or false (Wason, 1972, p.173). It was found that only 4% of the subjects selected E and 7 suggesting that the majority could not solve this simple reasoning task about a material conditional.

A range of subsequent experiments with similar material have shown a wide range of phenomena, for example, that people do arrive at the correct solution if the problem is presented in different forms (Stenning and van Lambalgen, 2008) and that even young children can arrive at the correct solution through extensive peer discussion (Moshman, 1999). It appears that the source of errors lies not in problems with logical reasoning but rather with those of interpreting the instructions in the terms intended by the experimenters. The literature on the selection task provides a microcosmic perspective on the issues that people have in applying their reasoning abilities to new domains of experience. These issues have been investigated more naturalistically in studies of the learning of advanced scientific and mathematical thinking skills (Tall, 1991; Gorman, 2005; Jonassen, 2007).

At the other extreme animal psychologists have investigated the reasoning capabilities of other species in terms similar to Piaget's stages (Hurley and Nudds, 2006; Crystal and Foote, 2009; Lurz, 2009). These studies provide insights into pre-linguistic human skills based on innate capabilities and learned reflexes, and on the types of problem-solving behavior that might be ascribed to them rather than to more advanced forms of reasoning. Studies of the *dual-mode* model of human reasoning have shown that we solve many problems through a wide range of *intuitive heuristics*, rarely resorting to explicit logical reasoning (Evans and Frankish, 2009). Evans (2008, p.256) notes that "Almost all authors agree on a distinction between processes that are unconscious, rapid, automatic, and high capacity, and those that are conscious, slow, and deliberative."

What are generally termed 'intuitive heuristics' can usually be modeled in rational terms even if they not generated through explicitly logical processes. For example, Gigerenzer and Brighton's (2009. p.108) example of how baseball player catches a ball, "fix your gaze on the ball, start running, and adjust your running speed so that the angle of gaze remains constant," describes a simple feedback loop readily modeled in logical terms. This situation-specific 'heuristic' can be derived from the general logic of the underlying dynamic principles, but that is not its foundation in practice. It should also be noted that the derivation from a specific model is a weak one in that the feedback loop makes the heuristic robust not only against 'noise' in the situation but also against the exact form of the dynamic equations; that is, the heuristic is more robust than the deeper theory (Weinmann, 1991). Powers (1973) discusses the application of feedback principles to achieve goals without any specific plan for their achievement in his model of *behavior as the control of perception*. The general principle is that in an uncertain world the strategy of using a sequence of short-term actions based on fast and simple inference with ongoing corrections is more robust than one based on longer-term complex plans.

Cherniak (1981, p.166) has analyzed the use of heuristics in terms of the minimal requirements for an agent to be deemed rational, "if A has a particular belief-desire set, A would undertake some, but not necessarily all, of those actions that are apparently appropriate." This is a very weak criterion not necessarily requiring any modeling or deductive competence on the part of the agent to perform very effectively in some situations. For example, Gaines (1971) has shown that there are control tasks in which a stochastic automaton with two states can outperform any finite automaton with any prescribed amount of memory and a complex strategy tailored to the task. That is, sometimes random guessing with no strategy other than a stopping rule for success is a better heuristic than any form of elaborate modeling and planning. Such results correspond to some effective modes of human behavior in difficult situations where rational modeling is proving ineffective, of just 'stirring the pot' and hoping that an opportunity to improve the situation will arise.

Such strategies rely on a multi-stage decision process where an error at any stage is not disastrous, that is, a robust environment that can be explored without undue risk. All animals modify their environments to create niches that enhance their chance of survival (Laland, Odling-Smee and Feldman, 2000), and humans have done so both by major physical modifications and through the development of cultures and institutions that support bounded rationality by providing *affordances* (Zhang and Patel, 2006) that make routine tasks simple. Such affordances may be modeled logically as *lemmas* that have the same role as they do in mathematical reasoning, of being usable without derivation. The source of the lemmas for individals may be, as in mathematics, not the derivation but instead the socio-cultural processes of belief propagation, and these processes may be tacit, imitative and through environmental affordances including symbolic media. Bourdieu (1989, p.86) has analyzed the *logic of practice* noting that it "not that of the logician" on the same grounds as those of Heidegger, but his corpus of work is valuable as a foundation for logical modeling precisely because it describes so many significant human phenomena which challenge logical modeling.

Sudoku puzzles have been recognized as providing test cases of pure deduction that "cannot be solved using pragmatic schemas or innate modules tuned to specific contents, or fast and frugal heuristics, or the atmosphere of premises, or probabilistic reasoning." (Lee, Goodwin and Johnson-Laird, 2008). The optimum computational solution is Knuth's (2000) *dancing links* algorithm for constraint satisfaction, but this is remote from human practice which is based on a small number of problem-solving strategies ranging from the very simple to the somewhat complex (Gordon, 2006). There are over 6x10E27 different Sudoku puzzles, and there are many variants of the basic puzzle involving additional constraints, so there is a wealth of experimental material. There has already been significant research in the constraint satisfaction community on the problem of assessing the difficulty of a given puzzle for humans based on the number and types of inference necessary (Henz and Truong, 2009). From a logical perspective, each solution strategy for Sudoku generates a consequence operator that can reduce the number of possible states of the puzzle, and set of such strategies generates a consequence operator that will solve some subset of the class of all Sudoku puzzles.

Piaget's focus on scientific knowledge and a related tendency in the literature on reasoning to associate it with the skills valued in the quantitative sciences has biased research on reasoning capabilities to use tasks that have well-defined mathematical and logical interpretations. The overall literature on the development of rationality is best viewed as providing a taxonomy of such tasks ordered in difficulty by the stage of intellectual development necessary to perform the task satisfactorily, on the assumption that the stage correlates with age in a culturally homogeneous cohort from the same species. Such a taxonomy of tasks is valuable educationally because it can be used to develop a progression of teaching materials such that satisfactory performance at one level indicates readiness to progress to the next. Teaching and training can then be managed effectively through performance-based progression without any knowledge of the internal processes of the student (Gaines, 1974; Gaines, 1997).

Much of the complexity of human reasoning seems to arise from its being resource-limited and situation-specific. Agents have to arrive at reasonable conclusions within memory and time constraints, and can exhibit a clear pattern of logical inference in one situation but fail to transfer it to a logically similar situation (Wason, 1972, p.191). A universal logic analysis might be based on multiple consequence operators indexed to subsets of the space of 'meaningful sentences' and weakening the closure constraint to one of partial closure. Such resource-limited local logics might be based on logical models of resource-bounded agents (Gabbay and Woods, 2008; Fisher and Ghidini, 2009) in the context of Barwise and Perry's (1983) *situation theory* and its later development (Barwise and Seligman, 1997). Cavedon (1998) and Bodanzo (2005) have discussed this, and Zalta's (1993) application of his intensional logic (Zalta, 1988) to model situation theory might provide a suitable logical framework. A recent special issue of the *Journal of Logic, Language and Information* provides a number of accounts of logical models of situated resource-bounded agents (Alechina and Logan, 2009).

5 High-level reasoning—the cognitive history of science

The domains of human reasoning which we might expect to exemplify the formal structures studied in universal logic are the deductive sciences for which it was developed, and their applications in the empirical sciences. The history of science developed as a major academic discipline in the twentieth century (Thackray and Merton, 1972; Viterbo, 2007) and literature of the detailed case histories of the logical and empirical processes of both major and minor scientists, their successes and their failures, is now vast; Cohen's (1994) historiography of the scientific revolution is a valuable survey. There has been a related growth of the literature on the social processes in scientific communities (Merton, 1973; Pickering, 1995), and on the cognitive processes of scientists and scientific communities (Giere, 1992; Carruthers, Stich and Siegal, 2002).

There are other rich resources relating to high-level formal reasoning such as studies of advanced mathematical and scientific thinking (Gorman, 2005), complex scientific problem-solving (Jonassen, 2007), and the creation of scientific concepts (Nersessian, 2008). There are also anthropological studies where ethnographers have studied the cognitive, cultural and social processes of communities of scientists investigating, for example, the immune system (Charlesworth, 1989), high-energy physics (Traweek, 1988), and the nature of the moon as samples of lunar soil became available (Mitroff, 1974). Gorman (1992) has provided a detailed personal account of his own scientific activities to provide data for modeling scientific thinking.

Detailed accounts have been prepared of the intellectual processes leading to breakthroughs in the deductive science, for example, the proof of Fermat's last theorem (Singh, 1997; Mozzochi, 2000). Rips (1994) has studied the psychology of formal proof empirically, developed a model of the processes involved and made it operational in a computer program, *PSYCOP*.

Some general aspects of these studies are relevant to the role of logic. There is substantial disagreement as to whether there is a *logic of scientific discovery* as Popper (1959) termed it. Feyerabend's (1975) argument in *against method* that "anything goes" seems to characterize many, but not all, aspects of the historic record; for example, Newton's development of laws of motion to support his millenarianism by enabling him to calculate past positions of the planets in order to date biblical prophecies and thus predict the date of the second coming of Christ in order to prepare the temple of Solomon (Manuel, 1974; Dobbs, 1991). It is not the classic history of the father of modern science which has filtered out those aspects that do not fit our normative conceptions of scientific respectability. It illustrates that 'anything goes' in terms of motivation, but it also illustrates a deductive rationale that was eminently logical from Newton's perspective.

In addition, all the studies emphasize the socio-intellectual networks that are involved in scientific progress which were brought into prominence by Kuhn's (1970) *structure of scientific revolutions*. The cognitive processes of a scientific community create the models that lead to the recognition of the anomalies that Dewey (1910) characterized as creating the *gaps* triggering though processes and, while the thinking may be by individuals, it involves cognitive interaction with others in the community including the use of past products of that community. This has been studied as a process of *distributed cognition* (Giere and Moffatt, 2003; Harnad, 2005), involving the *division of labor* in scientific communities (Kitcher, 1993), and *group rationality* (Sarkar, 2007) in managing a *theory net* of inter-related scientific theories (Balzer, Moulines and Sneed, 1987). Valle (1999) has provided a detailed account of such *collective intelligence* in operation in her analysis of the growth of the life sciences in the Royal Society as a scientific discourse community from 1665 to 1965.

From a phenomenological perspective, Schütz (1945) has characterized such a community as a *lifeworld* creating its own notion of *reality*, often nowadays termed a *community of practice* (Wenger, 1998). Wolff (1976, p.162-163) has modeled the acceptance of the rules prescribing a role in such a community as *surrender* of individuality to *catch* the essence of the community's reality:

> "From the standpoint of the world of everyday life, the mathematician, as we often put it, lives in the 'world of mathematics', dealing with 'nonreal' elements, notably numbers, whose relation to 'real' things, to 'reality', is not part of his concern. Analogously for the logician. What makes our subject-object approach to this attitude misleading is the fact that the subject, the student of mathematics or logic—his or her individuality, including motives and attitudes—is irrelevant for our understanding; the only thing that counts is the pursuit, with its results and questions."

A scientific community is a society of agents specialized in their rational processes by the culture of that community, acting as a distributed system to carry out the empirical and deductive processes that develop and maintain the theory network of that scientific community. From a universal logic perspective the implementation of the consequence operator has become distributed across the agents, which is interesting to model in its own right, and the overall system may be treated from a *collective stance* as a unitary, compound epistemological agent (Gaines, 1994).

The history of science literature provides many detailed case histories of high-level human reasoning that might be modeled in universal logic terms. A particularly relevant corpus of material is that associated with the development of the axiomatic approach that led to Tarski's formulation. The concern that Frege expressed about Hilbert's (1902) proof of the consistency and independence of the axioms for geometry instantiates Wolff's *surrender and catch* model—Hilbert had caught the formal essence of geometry by surrendering to its logical axioms—Frege felt that the surrender of any link between the axiomatic explicatum and the geometric went too far (Resnik, 1974). Tarski's formulation of the nature and definition of the logical supports Hilbert's stance, and his analysis of truth requires only a very weak link between explicatum and explicandum that leaves open the possibility that further conceptual analysis will result in additional connotations and associated axioms. A universal logic approach could model the relationship between the consequence operators resulting from incorporating successively richer connotations of the explicandum in the explicatum.

There are also interesting aspects of the later controversy over Gordan's recommending rejection of Hilbert's (1890) paper submitted to *Mathematische Annalen* generalizing Gordan's earlier results by proving the existence of finite generator for every ideal of ring of multivariate polynomials over a field. Gordan noted “das ist keine Mathematik, das ist Theologie!” (Noether, 1914, p.18), perhaps by analogy with the various *ontological arguments* for God's existence. Klein accepted the paper and Hilbert (1893) later developed a constructive proof. Some six decades later Gödel (1958), as part of his continuation of Hilbert's program to establish rigorous foundations of mathematics published in *Dialectica* an interpretation of intuitionistic arithmetic in a quantifier-free theory of functionals of finite type that can be used to develop constructive proofs of significant theorems (Feferman, 1993). Herz (2004) brought the historic wheel a full circle by using the *Dialectica* interpretation to translate some non-constructive proofs of the basis theorem to constructive ones. Fisher‘s (1966; 1967) studies of the social structure of research on invariants both before Gordan's results and after Hilbert's in terms of the “death of a mathematical theory” provides data on the social network structure and processes of distributed cognition involved.

The historic accounts of these controversies provide details of the human arguments about the legitimate development of the deductive sciences which led up to the development of the universal logic abstraction, and both the metalogical issues and the debate about them should be analyzable in logical terms. The historical accounts of the logical and mathematical issues under debate are already expressed in logical terms. The accounts of the debates about the issues show strong emotions were involved, such as Gordan's comments on Hilbert and Frege's (1979) introduction of comparisons with astrology and alchemy in his discussion of non-Euclidean geometry. However, as noted in Section 2, emotions have a rational basis, and McLarty's (2010) reconstruction of the Gordan-Hilbert issues places them in a logical context as does Blanchette's (2007) reconstruction of the grounds for Frege's concerns. It would be interesting to model both the external structure of the metalogical debate and the internal structure of the theories under debate in universal logic terms, and even more interesting to model the relationship between the two.

Kelly's (1955) psychological model of the role of *core constructs* as the foundations of an individual's persona, where the threat of change leads to a range of emotional behaviors, provides a logical framework within which to deconstruct the tacit assumptions that have been violated. It is the essence of Kuhn's (1970) analysis of the process underlying scientific revolutions that the tacit assumptions of an existing paradigm become seen by some as an impediment to progress but any attempt to change them is resisted by the community on what appear, in retrospect, to be irrational grounds. In metamathematics Frege's *Begriffschifft* provided the material for a revolution, but Hilbert's *Grundlagen* took it far beyond what he intended, and Tarski's axiomatization of *logical consequence* consolidated the new paradigm.

6 Logical models of reasoning in psychology

There have been many attempts to develop practical logics emulating or supporting reasoning, such as Rip's (1994) *PSYCOP*, Portoraro and Tully's (1994) *Symlog*, or Braine and O'Brien's (1998) *mental logic*. The problem with these and similar models as test material for a universal logic approach to human rationality is that they take for granted the notions of consequence and the definitions of logical constants of standard logic: *PSYCOP* because it is intended to model human approaches to deductive proof; *Symlog* because it is designed to support humans creating such proofs; and *mental logic*, even though it attempts to be naturalistic (O'Brien, 1995), because it presupposes that people use the standard logical constants that can represent concepts definitions and rules even though these are problematic in everyday human reasoning.

This issue of concept representation is one of the most widely studied aspects of human rationality—to what extent do human concepts resemble logical predicates? If we are to take logic as a normative ideal for human reasoning it is important to base its use on the logical form of the conceptual structures that people actually use. Mathematics provides one model where concepts are formally defined through their necessary and sufficient properties, and the ground set of undefined, primitive properties is minimized. Euclid's geometry is the archetype of this form of conceptual structure, and became the role model for both the deductive and empirical sciences (Netz, 1999).

It was natural in attempting to model human rationality in general to assume that all concepts are similar to those in mathematics and are defined through necessary and sufficient conditions. However, Waismann (1945) in a symposium on whether scientific concepts should be defined through their verifiability conditions argued that most concepts were essentially 'open' in that their definitions were "always corrigible or emendable." Weitz (1977) applied Waismann's notions to aesthetics in his book the *opening mind* and his analysis has come to be accepted in the philosophy of art. Hart (1961) applied them to the concept of law noting that the "open texture" of legal rules was advantageous in allowing them to be interpreted in situations that their authors has not foreseen. Conversely, mathematical concepts are not open and can be uniquely characterized as having no connotations other than their definitions (Tharp, 1989), although the choice of concepts used to represent a mathematical problem may be 'open' in a different sense (Cellucci, 1985).

The open nature of human concepts has been confirmed empirically in developmental psychology (Smith and Medin, 1981; Keil, 1989), anthropology (Rosch and Lloyd, 1978) and scientific practice (Nersessian, 2008). Studies of the conceptual frameworks and terminology of scientific teams working closely together and co-publishing show that major concepts in the discipline are being used in very different ways, but this phenomenon is not a topic of overt discussion (Shaw and Gaines, 1989). What seems most remarkable in the so-called 'exact sciences' is the capability to 'muddle through' with inexactly cross-calibrated concepts and yet still achieve significant advances (Fortun and Bernstein, 1998). One interesting aspect of this research is the finding that the notion of there being a definitional 'essence' to a concept, even if the properties of that essence are unknown, seems prevalent and originates in childhood (Gelman, 2003).

From a universal logic perspective, the open nature of all but mathematical concepts can be modeled through concepts having necessary but not sufficient properties. This corresponds to the minimality requirement of the normal structural definition of *conjunction* (Koslow, 2007) being lacking, so that the conjuncts are necessary consequences of the concept as a proto-conjunction but are not sufficient to form its minimal definition. At first sight knowledge structures where all concepts are essentially primitive may appear to have no capability to support reasoning, but abductive inference can be used to achieve many of the inference patterns normally attributed to definitions, has a formal logical structure (Aliseda, 2006), and has been proposed as the basis of practical reasoning (Gabbay and Woods, 2005). Gaines (2009) has demonstrated that knowledge structures of primitive concepts used with abductive inference processes can model many aspects of human reasoning in the arts and sciences, and that scientific definitions and normal deductive reasoning emerges naturally when the open concepts are artificially closed

A model of human thought processes that models the nature of open concepts is Kelly's (1955) axiomatic formulation of *personal construct psychology* based on Dewey's (1910) pragmatic instrumentalist analysis of *how we think*. Kelly was a minimalist who distilled Dewey's discursive account into a single axiom and eleven corollaries; he presents his model as a cybernetic one of how individuals construct their worlds by fitting a network of *templets* to experience. His model influenced Quillian (1967) in his development of *semantic networks* which evolved into *description logics* (Baader, Calvanese, McGuinness, Nardi and Patel-Schneider, 2003). It is attributed by Frake (1969) as the source of the notion of *contrast sets* in anthropology which were then used by Kuhn to model the formation of concepts as Wittgensteinian *family resemblances* (Andersen, 2000). It has been widely used in a range of human sciences including clinical psychology, education, and management science. It was adopted as a cognitive framework for the development of *knowledge engineering* tools by the expert systems community (Boose, 1986; Gaines and Shaw, 1993). It has been used to model bounded rationality in consumer behavior (Earl, 1986). It has been promoted as a preferred alternative to rational choice theory in *knowledge economics* (Loasby, 1999).

Kelly (1955, p.50) takes *anticipation* as the generative principle underlying all psychological phenomena, that "a person's processes are psychologically channelized by the ways in which he anticipates events," and deriving all other aspects of psychological processes as corollaries of this fundamental postulate. His first corollary is that of *construction*, that "a person anticipates events by construing their replications," and he provides a nice exposition of this in terms of the fitting of *templets*:

> "Man looks at his world through transparent patterns or templets which he creates and then attempts to fit over the realities of which the world is composed. The fit is not always very good. Yet without such patterns the world appears to be such an undifferentiated

homogeneity that man is unable to make any sense out of it. Even a poor fit is more helpful to him than nothing at all."

Kelly's use of the word 'anticipation' encompasses: *prediction* of what may happen; *action* to make something happen; *creative imagination* of what might happen or be made to happen; and *preparation* for eventualities that may well never happen. Fitting a templet can be a Procrustean action of changing the world to fit the templet, not just a passive process of gauging whether the world fits the templet.

He develops a simple logical model of anticipation taking as primitive notions: *experience*, *templet*, and a relation between them of *fit*, such that a templet may be said to fit an experience, and examines the possible relations between templets. One possible relation is *entailment*, that if templet X fits an experience then it entails that templet Y will also fit that experience. This can be represented as a transitive relation, $X \rightarrow Y$. Another possible relation is *incompatibility* creating *opposition* or *contrast*, that if one templet fits an experience then another will not fit that experience. This can be represented as an intransitive relation, $X \nrightarrow Y$.

The semantic networks that can be represented by these two relations have been studied by Gabbay and Schlechta (2009b) as *defeasible inheritance diagrams*, and Gaines (2009) has shown that they are adequate to represent a wide range of significant generic knowledge structures such as determinables, contrast sets, genus/differentiae, taxonomies, faceted taxonomies, cluster concepts, family resemblances and graded concepts. The normal constructions for logical constants (Koslow, 2007) do not necessarily correspond to templets within the system, and hence there is no assumption that definitions and rules are being used.

As noted above, a knowledge representation system where all the concepts are primitive clearly has no power of anticipatory inference other than how the concepts relate to one another. It is the epitome of an analytic system with no synthetic capability, and captures the network of relations constraining the use of words in a language as studied in *linguistic field theory* (Reuning, 1941; Spence, 1961). Dewey and Kelly model anticipation by postulating that it is based on matching novel experience against past experiences represented within the templet network. Such inference is essentially abductive, that templets generated by abstracting past experience as construed through the network of existing templets, will be matched against new experience, and the best matches will be used to anticipate other templets that will be hypothesized to fit the new experience.

The overall knowledge structures generated in this process are interesting because there are strong structural constraints on them that serve to make reasoning tractable. Kelly requires that all oppositions are relative so that the pair of templets in opposition each entail a common superordinate templet that he terms their *range of convenience*, essentially the *universe of discourse* or *context* in which they may be applied. A collection of these contextual constructs entailed by a *frame* templet (Barsalou, 1992) represents the dimensions of the *psychological space* (Kelly, 1969) for experience relevant to that frame. The experience templets each entail the frame and also entail templets subordinate to some of its contexts; they are *schemata* in Bartlett's (1932) terminology.

Abductive inference can be represented deductively as an assertion that at least one of the templets in memory must fit the new experience (Aliseda, 2006), and this can be introduced as a meta-level assertion to replace the use of abduction. However, it does not imply the general capability to use a structural disjunction on the part of the person using abductive inference. Kelly models it simply through his *choice corollary*, in essence that people freely chose from among alternatives they see as viable, a notion that Cherniak (1981) also introduces in his model of *minimal rationality*.

The Dewey/Kelly model of human inquiry provides a natural bridge between the psychology and logic of human reasoning that can be modeled within a universal logic framework without logicist assumptions that the entire apparatus of standard logic is psychologically available. In particular, it would be interesting to represent the effect of Kelly's structural constraints within that framework. The natural logical structure within which to represent it uses the four-valued semantics proposed by Belnap (1977) and Dunn (2000), and the translation of this to a standard two-value logic is the basis of computer implementations. The paraconsistency of these semantics nicely captures the lack of *ex falso quod libet* in human reasoning, but creates a problem in modeling how people use the contradictions of failed anticipations to adjust their schemata and networks. Gabbay, Rodrigues and Russo (2008) have shown that the AGM revision theory may still be applied by replacing the notion of inconsistency with one of *acceptability*. This is an important insight psychologically because it also enables one to model changes to representation systems initiated by considerations other than inconsistency such as improved simplicity, the avoidance of certain constructions, and so on.

7 Evolution of rationality

The questions about the evolution of reasoning have been addressed in various literatures addressing how the reasoning processes of the mind evolved from the physical processes of matter (Delbrück, 1986; Donald, 1991). Paleontologists have used fossil remains to trace the evolution of the size of the human brain and the increasing use of tools, noting a major discontinuity some 2.5 million years ago that seems to have facilitated the development of something resembling modern cognition and language (Gibson and Ingold, 1993; Noble and Davidson, 1996). Wynn (1985) has analyzed the manufacture of stone tools in terms of Piaget's stage theory and concluded that the third stage representing a distinction between human and animal intelligence was reached between 300 thousand and 1.5 million years ago, and the fourth stage of abstraction could have been present but would not be evident through analysis of the tools.

Cognitive anthropologists have studied processes of reasoning in different cultures, often primitive ones, and have generally concluded that the logical processes in primitive thought are the same as those of non-primitive thought (Tyler, 1969; D'Andrade, 1995), in particular, that the anthropological tenet of *psychic unity* in the human species is justified (Loflin and Silverberg, 1978). Hallpike (1979, p.489) reports a wide range of empirical studies of primitive thought based on Piaget's stage theory and concludes "It is far more fruitful to regard primitive thought as based on an incomplete logic rather than on a different logic from that which we know."

Bloom (1981) attributed the surpassing of Chinese science by the later-developing Western science to the lack of a counterfactual conditional in the Chinese language restricting hypothetical reasoning. Au (1984) replicated and extended Bloom's studies, showed methodological flaws and illustrated that hypothetical reasoning was used in Chinese thought. Lloyd (1996) sees the major difference in the two cultures as arising from the Chinese failure to develop the axiomatic reasoning that originated in Euclidean geometry.

There are major methodological problems in the study of the evolution of reasoning. One is that anthropological studies take standard logic as a normative criterion for rationality and fit that logic to the phenomena they observe. The chapter on *culture, logic and thinking* in (Cole, Gay, Click and Sharp, 1971) explains the issues well and gives an account of the nature and management of artifacts in empirical research into human rationality. It also provides another example of what appears to be complex deductive reasoning, that the Kpelle tribesman play *Awari*, a two-player board game in the *Mancala* family where seeds are moved around a board with two rows of 6 depressions according to simple rules of play and capture. It is noted that expertise in the game seems to require long chains of logical reasoning far beyond those normally studied, and raise the question as to whether it might be played skillfully without deep logical inference. That question is worthy of investigation in a universal logic framework because it is foundational to all studies of human rationality. Awari has become one of the games used to test computer game-playing in regular tournaments, and there is now a substantial literature on playing strategies, including an exhaustive database of the 9E11 possible situations and optimal plays (Romein and Bal, 2003), and an analysis of the information necessary to make a good move illustrating the trade-off between a good representation and depth of reasoning (Daoud, Kharma, Haidar and Popoola, 2004).

Another major problem is that the written record only extends back to about 1600 BCE enabling us to study, for example, logical reasoning in Babylonian mathematics and astronomy (Robson, 2008) and its influence on the Greek sciences that we see as a foundation for our own (Jones, 1991), but evidence of prior cultures of deductive science have been lost (Neugebauer, 1952). It is amazing that any material on clay tablets and papyrus has persisted for nearly four millennia, and the saga of how much of Aristotle's work survived demonstrates the chaotic nature of the process (Laughlin, 1995). The work of other major thinkers of that era such as Anaxagoras is known only through fragments culled by historians from the comments of later writers (Gershenson and Greenberg, 1964), but fragments that display a quality of logical inference comparable to any later figures of the scientific revolution some two millennia later.

Technologies supporting human reasoning have also evolved over the millennia to ameliorate the resource-limitations of the human brain. The invention of writing overcame the dependence on human memory constraining the previous oral tradition (Havelock, 1982). The invention of the printing press facilitated the diffusion of knowledge to create a global memory and distributed processing system for human knowledge (Eisenstein, 1979). The invention of the digital computer is enabling the massive information overload created by the growth of recorded knowledge to be more effectively managed (Hobart and Schiffman, 1998). We are still in the early phases of the computer revolution and do not know yet how it will change our modes of reasoning. Google is the first of the search engines to have become an effective extension to the memory processes of many people and its *pagerank* algorithm seems to have a correspondence to the operation of human memory (Griffiths, Steyvers and Firl, 2007). *Social networking* through the Internet seems to be changing the nature of the lifeworld for many people (Zhao, 2007). However, information technology evinces sustained exponential growth in all its aspects generated by positive feedback loops in which the technology is the major factor in its own growth to an extent that has never previously occurred (Gaines, 2006), and predictions of its longer-term impact on the nature of our civilization are highly speculative (Kurzweil, 2005).

Models of the economic evolution of the world-system demonstrate the need for a new basis for economic growth since the process to date has resulted in hyper-exponential population growth (Korotayev, 2005). Our civilization's inference engine has been based on the processing brains of the human population enhanced by the amplification of that intelligence through the growth of technology. Population growth became unsustainable in the last century and yet the growth of knowledge needs to continue to support life on this planet. Wojciechowksi (1983) has argued that most of our current problems have been created by the growth of knowledge but can only be managed through its continued growth. The compensatory process addressing the decline of population growth in the 1960s seems to be the enhancement of human knowledge production through information technology. Developments in our understanding and support of human rationality through advances in logic and mathematics are part of this process.

8 Logical consequence from a perspective of human rationality

Taking an consequence operator with no *a priori* axioms as the starting point for universal logic studies reflects the fact that one or more of each of the standard structural axioms has been dropped or modified at some time in a range of systems that are still reasonably construed as deductive logics (Doŝen, 1993). In the context of the questions posed for this special issue it is interest to examine those axioms from a psychological perspective. One can ask whether the standard structural axioms that Tarski (1956b) proposed are also consistent with his remark that "The concept of logical consequence is one of those who introduction into the field of strict formal investigation was not a matter of arbitrary decision on the part of this or that investigator; in defining this concept, efforts were made to adhere to the common usage of the language of every day life." (Tarski, 1956a). He made this comment in the context of his model theoretic notion of consequence rather than his earlier axiom system, but it is interesting to see to what extent the axioms represent common usage if they are to be used in modeling human reasoning.

Tarski's (1956b) axioms for a logical consequence relation as refined by Łoś and Susko (1958) and used by Wójcicki (1988) are that C is a consequence operator on the powerset of subsets of a set S if:

$X \subseteq C(X) \subseteq S$ (1) inclusion

$C(C(X)) \subseteq C(X)) \subseteq S$ (2) closure

If $X \subseteq Y \subseteq S$ then $C(X) \subseteq C(Y) \subseteq S$ (3) monotony

Antonelli (2005, p.5) says of axiom (1) that "It's not clear in what sense a relation that fails to satisfy this requirement can be called a *consequence* relation." However, this axiom fails to take into account the strong metalogical distinction made colloquially between *premises* and *conclusions* in order to avoid circular arguments. As Aristotle (1901, para. 73a5) remarks "saying that if A is then A must be is a simple way of proving anything."

Malinowski (1990) developed *q-consequence* as a variant of Tarski's axioms in order to model "mathematical practice that treats some auxiliary assumptions as mere hypotheses rather than axioms." He drops axiom (1) and compensates by modifying axiom (2) while retaining (3):-

$Q(X \cup Q(X)) \subseteq Q(X)$ (2q) q-closure

If $X \subseteq Y$ then $Q(X) \subseteq Q(Y)$ (3q) q-monotony

Malinowski defines q-closure as an equality but this is derivable from (2q) and (3q).

One can represent a C operator in terms of a Q operator and recover the Tarksi axioms. Define:

$$C(X) \equiv X \cup Q(X) \quad \text{(cq)}$$

We can derive axiom (1) trivially from (cq). Axiom (2) follows from:

$$C(C(X)) = C(X \cup Q(X)) = X \cup Q(X) \cup \ Q(X \cup Q(X)) \quad \text{(2.1) from (cq)}$$

$$C(C(X)) \subseteq X \cup Q(X) \cup \ Q(X) \subseteq C(X) \quad \text{(2.2) from (2.1), (2q) and (cq)}$$

Axiom (3) follows from:

$$C(X) \equiv X \cup Q(X) \text{ and } C(Y) \equiv Y \cup Q(Y) \quad \text{(3.1) from (cq)}$$

$$Q(X) \subseteq Q(Y) \quad \text{(3.2) from } X \subseteq Y \text{ and (3q)}$$

$$C(X) \subseteq C(Y) \quad \text{(3.3) from } X \subseteq Y\text{, (3.1) and (3.2)}$$

Thus the axioms for C can be derived from those for Q and, hence, the axioms for q-consequence do not change the rationality constraints imposed on the notion of consequence by the Tarski axioms.

We can use q-consequence to model the distinction between premises and conclusions for a c-consequence by defining a Q operator in terms of the C operator in such a way that any ‘premises’ that occur in a set are removed from the consequences of that set:-

$$Q(X) \equiv C(X)\backslash P \text{ where } P \subseteq X \quad \text{(qc)}$$

where P is a set that we chose to regard as premises. Condition (2q) follows from:

$$Q(X \cup Q(X)) = C(X \cup C(X)\backslash P))\backslash P = C(X \cup C(X))\backslash P = C(C(X))\backslash P \quad \text{(2q.1) from qc and (1)}$$

$$Q(X \cup Q(X)) \subseteq C(X)\backslash P \subseteq Q(X) \quad \text{(2q.2) from (2) and qc}$$

Thus the axioms for Q defined as in (qc) can be derived from those for C with the Q operator excluding the premises from the conclusions, and the original C operator can be recovered from Q by the definition (cq), since $X \cup C(X)\backslash P = C(X)$, given that $P \subseteq X$.

The standard axioms would never be used in their normal applications to support circular arguments because the distinction between premises and conclusions would be made along the lines of (qc). However, their potential to do so indicates that the C operator does not quite capture common usage of the notion of consequence where the distinction between premises and conclusions is significant in avoiding circular inference. Malinowski’s q-consequence interpreted above nicely captures that distinction and would be a good replacement for the standard axioms since, otherwise, it imposes precisely the same logical constraints.

What is shown above is that one can drop the axiom of inclusion/reflexivity but can compensate by essentially reintroducing it in the axiom of closure/transitivity using Malinowski’s construction. There may also be reasons for actually weakening inclusion in order to model other aspects of naturalistic consequence relations. For example, if one considers the elements of the sets on which the consequence operator acts to be *information resources* then axiom (1) may be interpreted as stating that a resource once used continues to be available for further use which may be inappropriate if the operator can consume a resource, as in *linear logic* (Abramsky, 1993). This might be a reasonable way of modeling a reasoning agent *forgetting* some of the premises already asserted which is the basis of many fallacies as failures of rationality, but it also a common problem in practical reasoning since people have limited and unreliable memory capacity.

Ageron (2007) has surveyed research on logics in which the inclusion axiom is dropped and analyzed them in terms of *taxonomies*, categories without identities. Zhang and Zhou (2009) have developed a formal model of knowledge forgetting in S5. Krause and Béziau (1997) discuss axiom (1) as one of *autodeducibility* in mathematical logics showing how it arises from the definitions of the consequence relation in various frameworks and noting that it is unnecessary if one uses Malinowski's construction. Wansing and Shramko (2008) review some of the issues that have arisen from this construction, particularly the role of multi-valued logics in its representation, and make some significant connections to Belnap's and Dunn's paraconsistent four-valued logic (Dunn, 1976; Belnap, 1977) and hence also to relevant logics (Mares, 2004b).

Axiom (2) is problematic in defining consequence to be the transitive closure of all the logical inference processes that underlie it, whereas each of those processes could reasonably be seen as deriving 'consequences' in the common usage of the word. We need to consider this fine structure of inference processes in order to model both the resource and situated aspects of human reasoning. Each inference strategy is itself a consequence operator satisfying some axiom set. If the inclusion axiom (1) is satisfied by all the inference strategies then we can consider them as a set, rather than a semigroup. Such a set of consequence operators defines a single overall consequence operator with a set of conclusions generated by the free semigroup of all possible sequences of operators from the set. A subset of a set of consequence operators that generates the same consequence operator as the full set is a *basis* for the full set, but the other operators might not be redundant if they lead to shorter proofs.

Such factoring also allows us to model situated reasoning since the ability to apply a reasoning strategy in some situations but not others may be represented by there being different reasoning strategies indexed to situations. The notation used does not need to change if C is taken to be a set of consequence operators rather than a singleton, and the relation generated by C is taken to be that derived from all possible sequences generated by C. The situation becomes more complicated if resource limits are considered and costs are associated with each sequence, but the logical framework remains the same.

Axiom (2) captures the notion of transitivity of consequence that is often taken in the human reasoning literature as fundamental to any deductive process. However, it should be noted that the underlying inference operations are generally not transitive, B being a direct inference from A and C being a direct inference from B does not imply that C is a direct inference from A. Someone might not see C as a consequence of A until they have first derived B from A and then C from B. They would then accept that C is a consequence of A because the colloquial term *consequence* is generally accepted to be transitive; that is, a chain of consequences can always be aggregated as a single consequence. Thus transitivity is assumed to be intrinsic to the normal usage of the term consequence. Apparent failures of transitivity can arise if there are tacit assumptions in the inference of B from A that are invalid in the inference from B to C, but these are best modeled as failures of monotonicity.

Axiom (3), requiring the consequence operator to be monotonic, is appropriate to formal reasoning but, if one wishes to model the human process of coming to conclusions with insufficient information and retracting some of those conclusions if they are contradicted as more information is acquired, some weaker axiom, or axiom set, is required. While the emulation of human commonsense reasoning originated as a topic in research on artificial intelligence, and early attempts at its implementation in semantic networks failed (Brachman, 1985), Reiter's (1980) formal model of *default reasoning* demonstrated that some forms of nonmonotonic reasoning could be rigorously defined. Reasoning with insufficient information has its own rationality constraints; for example, that it should become monotonic when the information has become sufficient.

Gabbay (1985) analyzed nonmonotonic reasoning within the universal logic framework of the Tarksi axioms, suggesting that the major rationality requirement is that conclusions when added to the premises should not undermine themselves, and this condition has come to be termed one of *cautious monotony*:-

if $X \subseteq Y \subseteq C(X) \subseteq S$ then $C(X) \subseteq C(Y) \subseteq S$ (4) cautious monotony

From axioms 4 and 2 we can also derive:

if $X \subseteq Y \subseteq C(X) \subseteq S$ then $C(Y) \subseteq C(X) \subseteq S$ (4t) cumulative transitivity

and hence:

if $X \subseteq Y \subseteq C(X) \subseteq S$ then $C(Y) = C(X)$ (4c) cumulativity

A consequence operator satisfying inclusion, closure or idempotence, and cautious monotony has come to be termed *cumulative*.

The axioms for q-consequence may modified in a similar fashion to provide rationality criteria nonmonotonic reasoning with a distinction between premises and conclusions:

if $X \subseteq Y \subseteq X \cup Q(X)$ then $Q(X) \subseteq Q(Y)$ (4q) cautious q-monotony

and q-cumulativity can be derived:

if $X \subseteq Y \subseteq X \cup Q(X)$ then $Q(Y) = Q(X)$ (4qc) q-cumulativity

so that Malinowski's variant axioms are also suitable to represent constraints on nonmonotonic reasoning.

A range of further constraints that may be placed on the rationality of nonmonotonic reasoning have been investigated (Makinson, 2005). For example, *rational monotony* (Lehmann and Magidor, 1992) captures the intuition that if something is not disproved it should be possible to add it to the premises without undermining any existing conclusions:

if $C(C(X) \cup Y)$ is consistent then $C(Y) \subseteq C(X)$ (5) rational monotony

This seems to correspond to a pattern of human reasoning, "if you cannot see a reason not to do something then it is reasonable to do it," which has obvious problems if the reasoning process is incomplete. However, Stalnaker (1994) showed that where the preferences between defaults allow for multiple possible extensions then rational monotony does not necessarily apply. Other axioms resolving this issue have been investigated, and Kraus, Lemann and Magidor (1990) showed that they can be unified as conditions on preferences over the sets of possible models arising from insufficient information. Lehmann (2001) showed that these results could also be interpreted in terms of Arrow's (1959) *rational choice functions* which are the foundation of *social choice theory* in economics (Arrow, Sen and Suzumura, 2002). As one might expect, there are also close relationships with logical constraints on theory revision (Alchourron *et al*., 1985) that are leading to a unified logical framework for theory change and nonmonotonic reasoning (Makinson and Gärdenfors, 1991; Makinson, 1993).

The weakening of the Tarski axioms to encompass nonmonotonic consequence operators can be viewed as an exercise in the development of universal logic before that term was coined, and should be consolidated as part of that literature. Makinson's (2005) *bridges from classical to no-nomontonic logic* provides an excellent survey of the state of the art within what is essentially a universal logic framework. Gabbay, whose 1985 paper originated this approach, has extended it recently to study the resource-origins of nonmonotonicity (Gabbay and Woods, 2008), the complexity of cumulativity when the domain is not closed under finite unions (Gabbay and Schlechta, 2008), and the rigorous formulation of defeasible inheritance in semantic networks (Gabbay and Schlechta, 2009a). Much of the conceptual framework developed in this literature carries over to the monotonic case and provides interesting new perspectives that are relevant to studies in universal logic.

It should be noted that failure of monotony in deduction is described in common usage by many terms such as *guessing*, *hidden assumptions*, and so on. The criteria of rationality expressed in the Tarski/Malinowski axioms do seem to be implicit in common usage of the term 'consequence' in the language of everyday life, and when they are violated in some way we tend to have qualifiers that describe the nature of the violation such as *direct* and *indirect* consequences, *conjecture*, *assumption*, and so on. In particular, we often trace violations of monotony back to the assumptions that led to them because the failure of those assumptions provides significant insights into the problem-solving situation. The failure of monotony may be seen as generating a contradiction and, as with paraconsistent adaptive logics (Batens, 2007)`, we use such contradiction not to draw any conclusion we please but instead to modify our premises to remove it, not expecting such modification to be uniquely determined.

9 Extending the axioms for specific logics

Another direction of development of the Tarksi axioms is to strengthen them rather than weaken them by adding further axioms that capture the logical properties of additional conceptual structures. Since the axioms are intended to capture the minimal essence of what it is to be a logic, this direction has the problem, or merit, of subsuming virtually all the logical literature. However, in the context of human rationality, there are certain foundational structures that it are important to represent such as those of space, time, deontic reasoning about permission and obligation, and counterfactual reasoning involved in planning, design and imagination. This suggests that it is important to capture modal logic. There are also aspects of many logical systems that seem foreign to human inference such as *ex falso quodlibet*, unnatural consequences of material and strict implication, and intractability. This suggests that it is important to capture some aspects of *relevance logic* (Anderson, Belnap and Dunn, 1975; Mares, 2004a) and of the management of intractibility in *description logics* (Baader *et al*., 2003).

The series of axiomatizations of planar geometry from Euclid (Artmann, 1999), through Hilbert (1902) to Tarski (1999) is a good starting point for a universal logic analysis of both concepts of space and of the development of formal inference (Netz, 1999). Wallace and West (1998) provide an excellent account from a mathematical perspective, Lachterman (1989) and Magnani

(2001) provide deep philosophical, psychological and social analyses, and Bos (2001) reviews the impact of Descartés' introduction of an algebraic approach.

A universal logic framework for modal reasoning already exists through the modern axiom schemas, K, D, T, B 4 and 5, for $\Diamond$ and $\Box$, the lattice of different combinations of the schemas from K through to KT5, and the various interpretations of the modal symbols to represent temporal, epistemic, deontic reasoning, and so on (Chellas, 1980). There are already implementations of effective reasoning systems for modal logics based on encoding these schemas for the high-performance, general-purpose constraint satisfaction systems (SAT-engines) now available (Armando, Castellini and Giunchiglia, 2000; Armando, Castellini, Giunchiglia, Giunchiglia and Tacchella, 2005). These provide a model for the implementation of computer-based inference systems based on universal logic formulations that could be used to support human reasoning processes.

Description logics provide an important contribution to logical modeling of human rationality because they commenced as foundations for the semantic networks originally proposed as models of human memory and reasoning processes (Quillian, 1967), and have developed to become a formal theory of the tractability of decidable sub-sets of first-order logic (FOL), and the implementation and application of reasoning systems based on them. Early semantic network implementations were criticized (Woods, 1975) for their lack of well-defined logical semantics, and later implementations formalized these (Brachman, 1983). However, it was found that reasoning in virtually any knowledge representation system rich enough to be useful is intractable (Brachman and Levesque, 1984; Levesque and Brachman, 1987; Nebel, 1990). The research focus became one of factoring representation schema into a set of basic constructs, analyzing the tractability of inference for various combinations of these constructs (Donini, 2003), and designing inference systems that realize the lower bounds resulting from these analyses (Tsarkov, Horrocks and Patel-Schneider, 2007). The description logic community has used a model-theoretic framework to provide a precise definition of each of the representation constructs considered through extensional semantics (Baader, 2003). Semantic networks are still used to represent knowledge but have been formalized to be a visual language that maps precisely to the logical constructs of description languages (Gaines, 2009).

10 Addressing the questions

The five auxiliary questions in the call for papers may now be addressed within the framework already discussed.

Capacity for reasoning: The first questions posed are, "Do all human beings have the same capacity of reasoning? Does a man, a woman, a child, a Papuan, a yuppie, reason in the same way?"

The difficulties of studying this issue empirically have been addressed in Sections 2 and 3, and a synopsis of relevant studies has been provided in Sections 4, 6 and 7. The answers from the empirical literature provide substance for the negative answer that commonsense might dictate. There are major differences in the capacity for reasoning: developmentally, culturally, and idiographically. However, if one rephrases the question to ask whether a unified logical framework might encompass all these examples and provide characterizations of the similarities and differences between them, then it would be reasonable to answer in the affirmative. What one might expect from a universal logic perspective is that there is a common kernel of axioms intrinsic to all reasoning and that individual differences may be modeled through families of additional axioms associated with different conceptual frameworks, and, to a lesser extent, by differences in resource constraints such as memory and time.

Evolution of reasoning: The second group of questions posed are, “Does reasoning evolve? Did human beings reason in the same way two centuries ago? In the future will human beings reason in the same way? Did computers change our way to reason? Is a mathematical proof independent of time and culture?”

A synopsis of relevant studies has been given in Sections 5 and 7. The time-scale of evolution of the human brain is far longer than that of recorded history, and the rationality exhibited in the literature of civilizations going back at least some four millennia shows no evidence of change in reasoning skills at the level of the kernel axioms. There have obviously been massive expansions of the conceptual frameworks that extend that kernel with axioms for more specific domains. This corresponds to the explosive growth of the *knowledge construct* (Wojciechowski, 1983) of which growth in our understanding of the basis of our own rationality has been part, but the logical kernel of that rationality has been constant if only because it captures the essence of what we mean by the term rational. Technologies to overcome the resource limitations of human reasoning have themselves evolved from writing through to computers, their effects may be modeled as changes in our modes of reasoning, and the impact of such technology is growing and has always been difficult to predict. As discussed in Section 5, the acceptable modes of mathematical proof have also changed, generally amidst much controversy; the issues of opaque computer proofs will likely be an ongoing one (Fritsch and Fritsch, 1998).

Situated reasoning: The third group of questions posed are, “ Do we reason in different ways depending on the situation? Do we use the same logic for everyday life, physics, economy?”

All the empirical studies reported indicate that human reason is highly situated not only in everyday life but in every sub-discipline of science. This can be seen as a complexity-theoretic necessity. The only way to cope with the combinatorial complexity of reasoning from first principles is to add lemmas as axioms, but the stock of lemmas in play has to be strictly limited for the same reason. So we divide the world of rationality into specialist subdivisions and further divide these as necessary to enable reasoning to take place. We divide to conquer but our conquest is of an increasingly large number of smaller domains. We can avoid complete fragmentation by managing less detail at higher level nodes, can hybridize between related nodes, and use analogical reasoning between unrelated nodes with similar structures. It is a paradigm that has worked well for a few thousand years but we do not know it is sustainable in the long term, and we not yet have any way of moving beyond it or of modeling its significant limitations.

However, within each sub-discipline it can be argued that the same kernel of logical principles apply, and the same human resource limitations apply. The pure mathematician developing a new theory is limited by her or his memory and needs to use much the same memory aids as someone preparing a household shopping list or a new design for an automobile. There is a need for precision of inference in all three situations, errors will be made, will have significant consequences and will be corrected. The domain-specific heuristics that make someone street-wise in mathematics, domestic science or car design come into awareness only if they fail; and, if they all fail, the agent will be at a loss as to how to proceed regardless of the nature of the activity. The final outcomes in the published theorem, the preparation of meals and the manufacture of a car will show little evidence of the reasoning processes that led to them.

Diversity of reasoning systems: The fourth question posed is, "Do the different systems of logic reflect the diversity of reasonings?"

Sections 8 and 9 have addressed the way in which the axioms need to be targeted on particular modes of reasoning. One may view the process whereby a kernel of axioms is extended with axioms characterizing the conceptual structures of different problem domains either as the development of different systems of logic, or as the addition of lemmas to the kernel logic. Historically, both processes are apparent but the most powerful tendency has been towards unification. Euclid brought planar geometry into a single unified system. Aristotle did so for syllogistic logic. Newton did so for dynamical systems. Frege did so for the foundations of mathematics and Hilbert extended that approach to major areas of mathematics. Kripke did so for the foundations of modal logic and hence unified many divergent developments in temporal, epistemic and deontic logics. Turing unified the reasoning processes for all logics within a single framework, and Cook unified issues of the resources required for reasoning within complexity theory. Eilenberg and Mac Lane unified mathematics and logic within the framework of category theory. A recent example noted in Section 8 is Lehmann's unification of current models of the axioms of nonmonotonic reasoning based on preference relations with the theory of social choice developed some fifty years earlier.

The metamathematical framework represented by universal logic has been incorporated in many major areas of logical development and is part of this tendency to unification. Ultimately, as we implement it in computers supporting logical frameworks (Harper, Honsell and Plotkin, 1993) that are programmed to achieve the best possible inference performance within the complexity bounds, and to manage libraries of lemmas and sub-theories to provide practical reasoning engines within specific domains, we may be able to support the need to divide and conquer through specialization through the availability of general-purpose reasoning engines applicable to all specializations. In terms of current technologies, the strategy of dividing into specializations in order to conquer complexity limitations will still be necessary as even quantum computation imposes such limits (Bernstein and Vazirani, 1997). We can see them as intrinsic to any attempt to model a combinatorially complex universe, but one that has allowed a divide and conquer strategy to be effective—at least, up to now.

Absolute basis of reasoning: The fifth question posed is "Is there any absolute true way of reasoning?"

The situated nature of reasoning not only in everyday life but also in the sciences suggests a negative answer to this question, but if we construe the question more in terms of the possibility of a unified model of reasoning based on a science of consequence operators then the answer might reasonably be positive. One can envision a sequence of axioms from minimal rationality constraints through multiple, situated, distributed and bounded consequence operators, through principled monotonic reasoning, to the standard Tarski axioms, to description logics, relevance logics, modal, intuitionistic and standard first order logics, to higher order logics, that might provide a taxonomy of rationality within a universal logic framework. Focusing on the consequence operators and introducing the logical constants uniformly through structural constraints allows the essential core of rationality to be studied uncluttered by the usual proliferation of domain-specific symbolism.

11 Conclusions—challenges for universal logic

A major theme of this paper has been addressing the degree of adherence of Tarski's formal definition of logical consequence to the phenomena of everyday life. Using the terminology of Kant and Carnap quoted in section 3 we may rephrase the question as "does the precisification of the explicandum of the folk notion of 'consequence' by the metamathematical explicatum of 'logical consequence' capture enough of the 'real essence' to be useful in the human sciences and does further bridging between the two provide a fruitful challenge to the logical sciences." I believe there is enough substance in the material surveyed in this paper to justify an unqualified 'yes' answer to that question and, hopefully, enough pointers to relevant approaches and literature to be useful to those pursuing some aspect of it as a research topic.

I have used the Tarski axioms in this paper to provide a concise focus for discussion, but, for many purposes, the Gentzen sequent formulation would be better, for others model theory and for others proof theory. Modeling the contexts in which reasoning takes place generally requires use of the other Bourbaki mother structures and makes the category-theoretic unification of these together with those for logic attractive. The problem, as always, is to do this without massive intellectual overload or opaque symbolism that creates a barrier to those in the applied sciences who might find such a framework useful. Hopefully, this paper may provide something of a bridge between empirical studies of human rationality and logical models of the rationality construct.

A number of research issues where a universal logic approach might be constructive have been mentioned in various places in this paper. That is, a focus on the nature of the consequence operators involved and their axiom schemas may clarify the issues in a particular research field, and may also advance universal logic research. In conclusion, here is a brief summary of the main challenges posed:

1 Provide a taxonomy of different notions of rationality as additional axioms constraining a consequence operator (Section 3);

2 Model Carnap's precisification, Piaget's equilibration processes and nature of theory-ladeness within a universal logic framework as ones of evolutionary theory change (Sections 4 and 5);

3 Develop a universal logic analysis of the chain of emergence of rational phenomena from those of the constraints of physical world such as causality and object identity, to their internalization through innate and learned reflexes, through to the reflective metalogical explanation of this entire process (Section 4);

4 Model societies of logical agents and the emergence of social rationality within a universal logic framework (Section 4);

5 Develop a universal logic analysis of multiple consequence operators, situated reasoning, distributed reasoning and bounded rationality (Sections 4 and 8);

6 Model the rational origins of intuitive heuristics and their interaction with reflective reasoning within a universal logic framework (Section 4);

7 Model reasoning processes taking place within a feedback loop of inference, action and assessment, and the robustness of such processes to uncertainty within a universal logic framework (Section 4);

8 Model human strategies for games that apparently involve only pure deductive reasoning such as those in the Sudoku and Mancala families as a set of consequence operators, and study how apparent deep reasoning might arise out of memory processes and reactive behavior (Sections 4 and 7);

9 Model controversies in the history of the development of the deductive sciences, externally in terms of the logic of the arguments in the controversy, internally in terms of the formal derivations in the published result, and holistically in terms of the relationship between them (Section 5);

10 Axiomatize the Dewey/Kelly model of human inquiry, knowledge representation, abductive anticipation and conceptual change as extensions of the nonmonotonic reasoning axioms (Section 6);

11 Consolidate developments in nonmonotonic reasoning, social choice and belief revision as a principled weakening of the Tarski axioms for a consequence relation (Section 8);

12 Express the axioms of geometry, modal logic and relevance logic as extensions of the Tarski axioms for a consequence relation (Section 9);

13 Axiomatize description logics, decidable fragments of first-order logic appropriate to knowledge representation, as extensions of the Tarski axioms for a consequence relation (Section 9);

14 Consolidate research on computer-based inference engines for the logics analyzed in challenges 10 through 13 to design an inference engine for universal logic with a plug-in architecture for various axiom systems (Section 10).

15 Communicate all this in such a way as to engage both the logic and human science communities.

Acknowledgements

I am grateful to Jean-Yves Béziau for the stimulating call for papers that provoked the writing of this paper, and to the anonymous referees for their perceptive and helpful comments.

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