## On the Relationship between Repertory Grid and Term Subsumption Knowledge Structures: Theory, Practice and Tools

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A number of practical knowledge acquisition methodologies and tools have been based on the elicitation and analysis of repertory grids. These result in frames and rules that are exported to knowledge-based system shells. In the development of repertory grid tools, the original methodology has been greatly extended to encompass the data types required in knowledge-based systems. However, this has been done on a fairly pragmatic basis, and it has not been clear how the resultant knowledge acquisition systems relate to psychological, or computational, theories of knowledge representation. This paper shows that there is a close correspondence between the intensional logics of knowledge, belief and action developed in the personal construct psychology underlying repertory grids, and the intensional logics for term subsumption knowledge representation underlying KL-ONE-like systems. The paper gives an overview of personal construct psychology and its expression as an intensional logic describing the cognitive processes of anticipatory agents, and uses this to survey knowledge acquisition tools deriving from personal construct psychology.

#### **1 PERSONAL CONSTRUCT PSYCHOLOGY**

George Kelly was a clinical psychologist who lived between 1905 and 1967, published a two volume work (Kelly, 1955) defining personal construct psychology in 1955, and went on to publish a large number of papers further developing the theory many of which have been issued in collected form (Maher, 1969). Kelly was a keen geometer with experience in navigation and an interest in multi-dimensional geometry. When he came to formalize his theory he took as his model Euclid's *Elements* and axiomatized personal construct psychology as a *fundamental postulate* together with eleven *corollaries*, terming the primitives involved *elements* and *constructs*. Kelly presented his theory as a *geometry of psychological space* (Kelly, 1969), and his conceptual framework is very clear if seen in these terms.

What Kelly achieved through the use of geometry was an intensional logic, one in which predicates are defined in terms of their properties rather than extensionally in terms of those entities that fall under them. Logics of knowledge and belief are essentially intensional (Hintikka, 1962), and in his time there were no adequate formal foundations for intensional logic. It was not until 1963 that Hintikka published the model sets formulation that gave intensional logic its *possible worlds* formal foundations (Hintikka, 1963), and hence formal foundations for cognitive science in logical terms only became possible in the late 1960s. The intensional nature of semantic networks in artificial intelligence was recognized in the late 1970s (Woods, 1975; Brachman, 1977; Shapiro, 1979), and their philosophical and logical structure as cognitive models has been detailed by Zalta (1988).

The dichotomous aspect of constructs is the most significant aspect of the difference between Kelly's constructs and current usage of the term, 'concept.' His *dichotomy corollary* states this (Kelly, 1955):

"A person's construction system is composed of a finite number of dichotomous constructs." (p.59)

and it is a consequence of the two-sided nature of a distinction represented in the geometry. That people tend to conceptualize the world in terms of restricted sorts that are then dichotomized is a phenomenon identified in antiquity (Lloyd, 1966) and common across many cultures (Maybury-Lewis and Almagor, 1989).

The taxonomic, abstraction, or subsumption, hierarchy between concepts is recognized in Kelly's *organization corollary* (Kelly, 1955):

"Each person characteristically evolves, for his convenience of anticipating events, a construction system embracing ordinal relationships between constructs." (p.56)

He uses this ordinal relation in the development of the psychology to model the dynamics of change in conceptual systems. For example, that one has "core constructs" that one is very reluctant to change because of the dependencies that exist within one's constructions.

Kelly's "repertory grid" methodology for eliciting conceptual structures has become a widely used and accepted technique for knowledge elicitation, and has been implemented as a major component of many computer-based knowledge acquisition systems. A comprehensive computer-based elicitation and analysis system for repertory grids was developed by Shaw with applications mainly in educational, clinical and management studies (Shaw, 1979). Gaines and Shaw suggested that repertory grids would provide a useful development technique for expert systems (Gaines and Shaw, 1980), and later published a validation study of the elicitation of the BIAIT methodology from accountants and accounting students using computer-based repertory grid elicitation (Shaw and Gaines, 1983). Boose, in an independent parallel study, reported success in a wide range of industrial expert system developments using computer elicitation of

repertory grids (Boose, 1984), and since then many knowledge acquisition systems have incorporated repertory grids as a major elicitation technique (Boose and Bradshaw, 1987; Diederich, Ruhmann and May, 1987; Garg-Janardan and Salvendy, 1987; Shaw and Gaines, 1987; Ford, Cañas, Jones, Stahl, Novak and Adams-Webber, 1990).

The repertory grid methodology has evolved in the light of application experience and now has major differences from that described by Kelly. Shaw took advantage of the processing power and interactivity of computers to introduce on-line analysis and feedback to the person from whom the grid was being elicited (Shaw, 1980). In expert systems terms, this can be seen as highlighting correlations that might be spurious and lead to incorrect rules in later analysis. Shaw and Gaines introduced new forms of analysis of the repertory grid based on fuzzy sets theory (Shaw and Gaines, 1979) which became the basis of rule extraction (Gaines and Shaw, 1986). Boose and Bradshaw made changes to the grid structure introducing hierarchical data structures to cope with more complex domains (Boose and Bradshaw, 1987). Bradshaw, Boose, Covington and Russo showed how many problems that did not seem appropriate to repertory grids could be formulated in terms of them (Bradshaw, Boose, Covington and Russo, 1988).

The original repertory grid methodology was based on only one aspect of Kelly's personal construct psychology, his dichotomy corollary. The standard grid is a flat structure of elements described in terms of dichotomous constructs that does not represent the hierarchical structure of Kelly's organization corollary. Hinkle developed a technique of *laddering*, based on "why" and "how" questions, for investigating ordinal relations between constructs (Hinkle, 1965), and Boose incorporated a laddering tool in ETS (Boose, 1986). However, ordinal relations between constructs were not the primary focus in initial applications of repertory grid tools.

This changed as the second generation toolbench, AQUINAS (Boose and Bradshaw, 1987), was developed in the light of experience with ETS, and hierarchical structures of tasks, experts, elements and constructs were introduced into the data structures and interfaces. It also changed as conceptual induction techniques were used to derive hierarchical concept structures from the rules extracted from repertory grids (Gaines and Shaw, 1992). Recently, the intensional logic underlying the psychological primitives of personal construct psychology has been developed in detail (Gaines and Shaw, 1990), and this has been used to develop knowledge acquisition tools based on a visual language that corresponds to a formal semantics for semantic nets (Gaines, 1991c). These later developments suggest that personal construct psychology can also provide foundations for tools in which ordinal relations are a primary focus, such as those that use some form of semantic network to build domain and task ontologies directly.

#### 2 THE INTENSIONAL LOGIC OF PERSONAL CONSTRUCT PSYCHOLOGY

Kelly's geometrical model of personal construct psychology may be reformulated as a corresponding intensional logic of knowledge representation. We take his notion of a distinction as primitive and examine how distinctions may relate to each other in psychological space. If one distinction carves out a region that contains that carved out by another then the first distinction may be said to *subsume* the second. If one distinction carves out a region that does not overlap that carved out by another then the first distinction to the second. If one distinction may be said to be *disjoint* to the second. These relations are in themselves sufficient to define an intensional logic of distinctions in that the more complex relations may be composed from them. Extensional considerations may be introduced by noting that, if an element is placed within the region carved out by a distinction, then we may say that the distinction is *asserted* to apply to the element.

The subsumption and disjoint relations may be defined in an algebraic formalism by representing distinctions by bold lower case letters such that a distinction applied to another distinction is concatenated to the right of it. Then the definition above translates as one distinction will be said to *subsume* another if it can always be applied whenever the other can. It can be represented formally as:

"b subsumes a" 
$$a \rightarrow b \Leftrightarrow + xa \Rightarrow + xb$$
 (1)

That is, **b** subsumes **a**, if and only if whenever one asserts **xa** one also asserts **xb**. The definition is to be read intensionally in terms of a *commitment* to the way in which distinctions will be made, such that if **a** is made then there is a commitment to **b** being made also. This is why the form  $\forall x$  is avoided—the notion of all the distinctions to which **a** and **b** may be applied is not well-defined.

Subsumption corresponds to increasing generality since the subsuming distinction can be applied to at least as many things as that subsumed. In (1) concept  $\mathbf{a}$  is said to be *subordinate* to concept  $\mathbf{b}$ , and  $\mathbf{b}$  *superordinate* to  $\mathbf{a}$ . Subsumption supports Kelly's organization corollary, and captures his use of the term that one construct subsumes another, and also the use of the same term in knowledge representation, that one concept subsumes another. Subsumption between computational concepts corresponds to the "is-a" relation in knowledge representation schema. The interpretation of subsumption in terms of commitment above corresponds to the definitional form of the "is-a" relation. The computed form of "is-a" requires some further structures which are developed in the next section when primitive and non-primitive concepts are differentiated.

The disjoint relation is definable in similar terms, that one distinction is disjoint with another in that one can never be applied whenever the other can. It can be represented as:

"a disjoint b" 
$$\mathbf{a} \rightarrow \mathbf{b} \Leftrightarrow \mathbf{i} \mathbf{x} \mathbf{a} \Rightarrow \neg \mathbf{i} \mathbf{x} \mathbf{b}$$
 (2)

That is,  $\mathbf{a}$  is disjoint with  $\mathbf{b}$ , if and only if whenever one asserts  $\mathbf{xa}$  one does not assert  $\mathbf{xb}$ . The definition is again to be read intensionally in terms of a commitment to the way in which distinctions will be made, such that if  $\mathbf{a}$  is made then there is a commitment to  $\mathbf{b}$ not being made. Disjoint is a symmetric, intransitive relation over distinctions, and supports Kelly's dichotomy corollary and the definition of disjoint concepts in knowledge representation.

It is interesting to note that definition (2) is an asymmetric definition of what is clearly a symmetric relation. Logically, this is possible because the reverse implication can be derived from (2), that is, if one asserts **xb** one cannot assert **xa** because that would imply  $\neg + \mathbf{xb}$ . This derivation of symmetry from asymmetry may be logically simple, but it is not semantically trivial. In terms of knowledge representation it corresponds to the essential sequence of definitions: if we define **a** first we cannot define it to be disjoint with **b** because **b** is not yet defined. Psychologically, this asymmetry appears to be related to the empirical asymmetries Adams-Webber has observed in the use of the, apparently symmetric, poles of a construct (Adams-Webber, 1979).

The  $\rightarrow$  and — relations are complementary in establishing four possible binary relations between distinctions, that  $\mathbf{a}\rightarrow\mathbf{b}$ ,  $\mathbf{b}\rightarrow\mathbf{a}$ ,  $\mathbf{a}-\mathbf{b}$ , or none of these. The two subsumption relations can hold together giving an equivalence relation on distinctions. The disjoint relation is incompatible with the subsumption relations, and is *inherited* through subsumption, that is:

$$\mathbf{a} \longrightarrow \mathbf{b} \text{ and } \mathbf{c} \rightarrow \mathbf{a} \Rightarrow \mathbf{c} \longrightarrow \mathbf{b}$$
 (3)

#### **3 A VISUAL LANGUAGE FOR THE LOGIC**

The arrow and line notion adopted in definitions (1) and (2) translates to a graphical notation defining a *visual language* for the logic (Gaines, 1991c). As shown at the top of Figure 1, Kelly's "construct" in psychological space can be represented by a pair of disjoint concepts corresponding to what he terms the construct "poles," both subsumed by a third concept corresponding to what he terms the "range of convenience." It is this fundamental conceptual unit, or templet that we fit over the world, being a pair of disjoint concepts applied to a restricted domain that characterizes Kelly's use of the logic as a foundation for cognitive psychology. In logical terms, he emphasizes the importance of *opposition* as relative negation applied within a context, rather than absolute negation free of any context. The psychological unit is the triple of concepts in the relation shown rather than the individual concept, or logical predicate, in isolation.

At the center of Figure 1, the abstract components of a concept are given specific instances to exemplify their application. "Evaluable" things may be classified into two disjoint classes, "good" and "bad."



# Figure 1 Representation of abstract and specific constructs and scales in a visual language for specifying definitions and assertions in the intensional logic

The emphasis on dichotomous concepts may give the impression that constructs are essentially binary in nature. However, at the bottom of Figure 1 is shown how Kelly's "shades of gray" arise naturally through the addition of related concepts compatible with the original dichotomy. The dichotomy has been split into two such that "bad" is now disjoint both from "good" and "fairly good", and "good" is now disjoint from both "bad" and "fairly bad." "Mediocre" has been added as an additional concept intermediate between "good" and "bad", defined as "fairly good" and "fairly bad." In tools such as the repertory grid these intermediate concepts are represented on a numeric scale as shown under the bottom structure of Figure 1.

The structures in Figure 1 are simple semantic networks in the style of KL-ONE (Brachman and Schmolze, 1985) or KRS (Gaines, 1991a), but they have well-defined logical semantics as defined above, and also strong psychological foundations in personal construct psychology. There is an analogy between the visual language and the representation of chemical structures as atoms and bonds. Distinctions are the atomic primitives in personal construct psychology, and further constructions may be seen as complex 'molecules' formed by distinctions joined through subsumption and disjoint 'bonds.' For example, the complex structure at the bottom of Figure 1 may be seen as the composition of two of the basic construct structures shown at the top. Figure 2 illustrates this with an example developed later in the paper.



Figure 2 Concepts defined in terms of others, and their application to representing anticipations as rules supporting inference

Multiple constructs in psychological space correspond to multiple axes of reference, and the planes representing their distinctions and ranges of convenience intersect to define regions of the space corresponding to conjunction, composition and multiple inheritance in the logic as shown at the top of Figure 2. This also illustrates an important distinction between the concepts defined by basic distinctions and those defined by intersections. The former are said to be *primitive concepts* and the latter non-primitive, or computed, concepts. In the visual language primitive concepts are distinguished by having a small internal horizontal line at their left and right edges. A primitive concept is incompletely defined in that we have complete freedom of choice as to where to place an element relative to the regions defining its distinction. However, no such freedom exists for non-primitive concepts since they are defined as the intersection of primitive concepts. Logically, we have to *assert* that a primitive concept applies to an element, whereas we can either assert that a non-primitive applies or *recognize* that it applies through the previous assertion of the primitives that define it. In knowledge representation this recognition is termed *classification* (Borgida, Brachman, McGuiness and Resnick, 1989).

The definition of subsumption in (1) applies to non-primitive concepts, but it is no longer a matter of direct commitment but rather of derivation from the composition of commitments for concepts defining the intersection. The "is-a" relation for non-primitive concepts is computable rather than definable—the commitment to their definition in terms of their structure entails a commitment to a derived, rather than a defined, "is-a" relation. Confusion about these two forms of concept, and associated "is-a" relations, caused problems in early developments of semantic nets (Brachman, 1983).

Kelly's theory of anticipation is based on attaching significance to such recognizable intersections:

"What one predicts is not a fully fleshed-out event, but simply the common intersect of a set of properties" (Kelly, 1955)

The logic remains intensional because there is no implication that elements have already been construed within the intersections. The attachment of an anticipation to the intersect corresponds to a commitment to place an element that falls in this intersect in the region defined by the pole of some other construct also. In logic this is a *material implication* rather than an entailment in that it is not necessitated by the way in which the distinctions are defined but is instead an auxiliary commitment or *rule*. Rules allow a cognitive system to be anticipatory in containing structures which from one set of distinctions made about an event will imply that others should be made leading to prediction or action. Rules play a similar role in computational systems in generating recommendations for decision or action. Overtly modeling the conceptual system of an expert as such a structure is a basis for emulating the expert's performance in a knowledge-based system.

As shown in Figure 2, Kelly's model of anticipation is represented in the visual language by an additional primitive, a rectangle with vertical bars, representing material implication or a rule. The rule in the center applies to a spatial mapping techniques example used later in this paper. It has the premise that if a technique is "Local" and involves "Linear interpolation" then the conclusion is that it is "Requires no model." At the bottom right of Figure 2, an individual "Hand contouring", represented in the visual language as a rectangle, is asserted to be "Local" and "Linear interpolation," represented by arrows from the individual to these concepts. When the entire knowledge structure of concept definitions, rules and assertions, is then compiled and run through the inference engine, the graph output is that shown at the bottom right of Figure 2. Hand contouring has been inferred to require no model.

The logic based on Kelly's axiomatic presentation of personal construct psychology, and the visual language representing it, both extend to support the additional features normal in term subsumption knowledge representation systems, such as attributes and relations, or "roles" as they have been termed generically (Brachman and Schmolze, 1985), rules with exceptions (Gaines, 1991b), and contexts (Sowa, 1984). Figures 1 and 2 have been presented in a graphing tool, KDraw, that provides a fully operational semantics for the input and output of knowledge structures in the visual language, and further illustrations of its application are given later.

#### **4 THE REPERTORY GRID**

Kelly introduces the "role repertory grid" (Kelly, 1955) as a means for investigating a person's conceptual structure relevant to inter-personal relations by having them classify a set of people significant to them in terms of elicited personal constructs. Figure 3 shows the general form of a repertory grid and its relation to the conceptual structures already discussed. If one takes a particular concept somewhere in the lattice, and a set of individuals asserted to fall under that concept, then the properties defining the concept generate distinctions about the individuals falling under that concept. These distinctions form the rows of a matrix, the individuals form the columns, and the constraints applying to a particular individual relative to a particular distinction form the values in the matrix.

In simple applications of the repertory grid these constraints are taken to be the values of the individuals on the roles corresponding to the distinctions. However, it is apparent from Figure 3 that concepts subordinate to those defining the scope of the grid may also be used as if they were individuals, and these may be expected to have more general constraints than single values. Hence in extended repertory grid elicitation, such as that of AQUINAS (Boose and Bradshaw, 1987) the 'values' in the matrix can in themselves be complex constraints.



Figure 3 The repertory grid as a matrix of concepts, individuals and constraints

Poquiros no model	5	5	1	1	2	1	4	1	5	2	5	Paguiras model
Requires no model		5	4	1	3	1	4	4	5	2	5	
Interval data	5	1	1	1	4	4	1	1	1	1	1	Nominal data
Non-polynomial	5	5	1	1	1	1	5	1	5	1	5	Polynomial
Global	1	1	3	4	4	4	5	2	1	4	1	Local
Intuitive	4	4	5	3	2	1	5	4	5	3	5	Mathematical
Requires spatial search	5	5	2	1	2	3	5	5	5	3	5	Does not require spatial search
Discontinuous	5	5	4	2	1	3	5	5	5	5	5	Continuous
Does not honour data	5	5	2	3	1	2	4	4	5	2	1	Honours data
Linear interpolation	5	5	2	2	3	1	5	5	5	5	5	Non-linear interpolation
Difficult to understand	2	4	1	4	4	5	1	2	1	4	1	Easily understood
Few points	1	3	1	5	3	1	3	2	1	4	2	Many points
Does not consider non-spatial attributes	2	2	2	2	3	3	1	1	2	1	5	Considers non-spatial attributes
	į	÷	÷		į	÷	÷	÷		÷.		Vector trend surface analysis
	÷.		÷		÷	÷	- {	÷	1	2.5		Negative exponential surface
	÷	;	÷	ł	ł	÷	÷	÷	5.			Most predictable surface
	÷.		÷		÷	÷	- {	÷.,				Double Fourier series
	÷	;	÷	ł	ł	÷	÷.,					Bicubic splines
	÷.			÷	÷	1						Hand contouring
	÷	;	÷	ł	4							Proximal mapping
	÷.			ί.			• • • •					Distance weighted averaging
	÷.	÷	÷						• • • •			Kriging
	÷,											Trend surface analysis
	<u>.</u>						• • • •		• • • •	• • • • •	• • •	Probability mapping

Figure 4 A repertory grid about spatial mapping techniques

Figure 4 shows a basic repertory grid elicited from a geographer about spatial mapping techniques. The mapping techniques used as elements are listed as column names at the bottom. The poles of the constructs elicited are listed on the left and the right as row names. The ratings of the mapping techniques along the dimensions of the constructs form the body of the grid. Figure 5 shows the constructs defined in Figure 4 exported to KDraw in the format of Figure 1. The tool used for the elicitation and analysis of grids, KSSO, also allows them to be exported to KDraw and shells such as NEXPERT and BABYLON, as attribute-value structures rather than conceptual primitives.



Figure 5 Spatial mapping techniques domain represented in the visual language

The psychological function of the repertory grid is to provide a technique for building the conceptual structure without direct elicitation of concepts and their structures and relationships. The assumption is that it may be easier for a person to provide exemplary individuals in the domain of interest, and then to state in fairly concrete terms how they would distinguish them in terms of properties relevant to the purpose of eliciting the grid. In terms of the intensional logic of the concept structure, the extensional specification of how concepts apply to individuals is clearly inadequate to fully specify the concept structure. However, the structure must be consistent with its model and hence it is possible through suitable analysis techniques to approximate the structure from the extensional data, as is discussed in the next section.

#### **5 CONCEPTUAL CLUSTERING**

In analyzing repertory grid data, distance measures play an important role in conceptual clustering and induction. In terms of the logic and visual language, there is a natural construction of a distance between two concepts,  $\mathbf{x}$  and  $\mathbf{y}$ , as shown on the left of Figure 6. Let  $\mathbf{u}$  be some minimal upper bound of  $\mathbf{x}$  and  $\mathbf{y}$  subsuming both of them, and  $\mathbf{l}$  some maximal lower bound subsumed by both of them, and  $\mathbf{U}$  be the extension of  $\mathbf{u}$ , and  $\mathbf{L}$  the extension of  $\mathbf{l}$  over some universe of individuals. If  $\mathbf{x}$  and  $\mathbf{y}$  are identical so will be  $\mathbf{U}$  and  $\mathbf{L}$ , whereas if they are disjoint  $\mathbf{L}$  will be empty. Hence a natural distance measure is the number of individuals that are in  $\mathbf{U}$  but not  $\mathbf{L}$ :

"x distance y"  $d(\mathbf{x}, \mathbf{y}) = \mathbf{C}\mathbf{U} - \mathbf{C}\mathbf{L}$  (4)

where CU and CL are the cardinalities of U and L respectively. This measure satisfies the triangle inequality and can be normalized by dividing by its maximum possible value, CU. It is clearly dependent on the universe of individuals involved, but this is appropriate to measuring concept distance in an extensional context. Intensional concept "distance" independent of context is reflected in the relational structures already developed.

The distance measure defined readily extends to dichotomous constructs through the comparison of poles as shown on the right of Figure 6:

"
$$\mathbf{b} - \mathbf{c}$$
 distance  $\mathbf{d} - \mathbf{e}$ "  $d(\mathbf{b} - \mathbf{c}, \mathbf{d} - \mathbf{e}) = \mathbf{C}\mathbf{A} - \mathbf{C}\mathbf{F} - \mathbf{C}\mathbf{G}$  (5)

This measure is a count of the numbers of individuals that fall under the opposite pole of the other construct. Note that it is not invariant if one construct is reversed. This construction generalizes to scales with more than three points. If these scales are numbered linearly it computes a "city block" distance measure—which is precisely that used in construct clustering algorithms such as FOCUS (Shaw, 1980). These distance measures enable natural clusters to be seen that may be grouped as part of a coherent concept, for example, in that they are all contributors to an evaluatory dimension.



Figure 6 Calculation of distance measures between concepts and between constructs

For example, Figure 7 shows a FOCUS analysis of the grid of Figure 4 in which the distance measure defined in (5) has been used to develop two matrices of inter-element and inter-construct distances. The sets of elements and constructs have then each been sorted to re-order the grid in such a way that similar elements and similar constructs are close together. Thus, near the bottom of the construct clusters, it can be seen that the dimension "Discontinuous—Continuous" is used very similarly to "Requires spatial search—Does not require spatial search", and that both of these relate closely to "Linear interpolation—Non-linear interpolation." Similarly near the top of the element clusters, "Probability matching", "Most predictable surface" and "Trend Surface Analysis" are construed as closely related techniques with very few distinctions between them.



Figure 7 FOCUS hierarchical clustering of spatial mapping grid

#### **6 RULE INDUCTION**

The measures used in the induction of a rule linking to concepts are also readily derived as shown in Figure 8. CX is the number of anticipations made by concept x as the left hand side of a rule, and CL is the number which are correct. Thus, the measures of the validity of inducing the rule,  $x \rightarrow y$ , are:

"prior probability of y" 
$$p(y) = CY/CU$$
 (6)

"probability correct 
$$\mathbf{x} \rightarrow \mathbf{y}$$
"  $p(\mathbf{x} \rightarrow \mathbf{y}) = \mathbf{C}\mathbf{L}/\mathbf{C}\mathbf{X}$  (7)

"probability by chance  $\mathbf{x} \rightarrow \mathbf{y}$ "  $c(\mathbf{x} \rightarrow \mathbf{y}) = I_{p(\mathbf{y})}(C\mathbf{X}-C\mathbf{L}, C\mathbf{L}+1)$  (8)

where I is the incomplete beta function summing a binomial distribution tail.

These measures are precisely those used by Induct (Gaines, 1989) in inducing rules from datasets. In the application to repertory grids Induct searches for potential rules whereby a target predicate may be deduced from some of the others, and constrains the search to rules whereby the probability that they arise by chance is less than some prescribed threshold. The basic search techniques have been well documented by Cendrowska (1987) but for practical applications they need to be controlled by these probabilistic measures, and also to be extended to generate rules with exceptions as these are both more compact and more in accordance with human practice (Gaines, 1991b).



Figure 8 Induction of rules between concepts

To illustrate rule induction from repertory grids,

Figure 9 shows an Induct analysis of the grid of Figure 4 in an attempt to determine the rules underlying the use of the term "model," which was a major source of conceptual and terminological difference between experts in the studies from which this data is drawn (Shaw and Gaines, 1989).

Points=Many points -> Model=Requires no model 100% 7.44%

Locality=Local & Interpolation=Linear interpolation -> Model=Requires no model 100% 7.44%

- Data type=Interval data & Type=Non-polynomial & Locality=Local -> Model=Requires no model 100% 7.44%
- Data type=Interval data & Type=Non-polynomial & Understanding=Easily understood -> Model=Requires no model 100% 7.44%
- Formality=Mathematical -> Model=Requires model 100% 4.23%

Search=Does not require spatial search -> Model=Requires model 100% 6.64%

Understanding=Difficult to understand -> Model=Requires model 100% 6.64%

Figure 9 Induct analysis of spatial mapping data

The first percentage at the end of each rule is the *probability correct* as defined in (7), and the second is the *probability by chance*, or statistical significance, as defined in (8). Figure 10 shows these rules exported from KSS0 to KDraw. The frame definition of Figure 5 and the rules of Figure 10, both derived from the grid of Figure 4, may be edited within KDraw and then exported to a knowledge-based system shell as an operational knowledge structure. Practical system development involves the derivation of such structures for the different sub-domains involved, together with the addition of rules that export inferences from one sub-domain to another.



Figure 10 Rules about which techniques require a model represented in the visual language

#### **7 USING REPERTORY GRIDS**

The use of the repertory grid to elicit concept structures involves a variety of psychological and analytical techniques, including:

- 1. Careful definition of the purpose of the elicitation and the appropriate sub-domain to be considered. Maintaining this context so that the purpose and domain do not tacitly change during elicitation is also very important.
- 2. Choice of exemplary individuals that characterize the relevant features of a domain. This choice is very important and is a major focus of attention in both tool design and application. Fortunately, experts often find it natural to discuss a domain in terms of stereotypical cases, but much care is required to elicit a full range of stereotypes adequate to characterize a domain.
- 3. Various techniques may be used for initial element elicitation including interviews, protocol analysis, brainstorming with groups of experts, and keyword extraction from relevant textual material (Shaw and Gaines, 1987; Shaw and Woodward, 1990).
- 4. Online analysis of the interim conceptual structures may be used to detect closely related distinctions and use this to request information on potential stereotypes that might specifically reduce the closeness of the distinctions (Shaw, 1980).
- 5. The elicitation of some initial distinctions may again derive from interviews, protocols, brainstorming and text analysis.
- 6. When no prior information is available, triadic elicitation in which a randomly selected set of three individuals is presented with a request to state in what way are two alike and differ from the third can be effective
- 7. Online analysis of the interim conceptual structures may be used to detect closely related individuals and use this to request information on potential distinctions that might specifically reduce the closeness of the individuals (Shaw, 1980).
- 8. The conceptual structure can be developed through various forms of hierarchical and spatial cluster analysis such as FOCUS (Shaw, 1980) and principal components analysis (Slater, 1976, 1977).
- 9. Rule induction may be used both to derive potential implications between concepts and also, since the premise of a rule is itself a concept, to develop non-primitive concepts and their subsumption relations (Gaines and Shaw, 1992).
- 10. Direct elicitation of the concept structure may be mixed with indirect development of the grid (Boose and Bradshaw, 1987; Gaines and Shaw, 1990).

#### **8 CONCLUSIONS**

Personal construct psychology is a theory of individual and group psychological and social processes that has been used extensively in knowledge acquisition research to model the cognitive processes of human experts. The psychology has the advantage of taking a constructivist position appropriate to the modeling of specialist human knowledge but basing this on a positivist scientific position that characterizes human conceptual structures in axiomatic terms that translate directly to computational form.

The repertory grid knowledge elicitation methodology is directly derived from personal construct psychology. In its original form, this methodology was based primarily on the notion of dichotomous constructs and did not encompass the ordinal relations between them captured in semantic net elicitation. However, it has been extended in successive tools developed for applied knowledge acquisition and tested in a wide variety of applications.

This paper has given an overview of personal construct psychology and its expression as an intensional logic describing the cognitive processes of anticipatory agents. A theoretical framework has been developed and shown to provide logical foundations for personal construct psychology and computational knowledge representation schema. The framework is generated from the single primitive of "making a distinction." It has been used to provide cognitive and logical foundations for existing knowledge acquisition tools and techniques, and for the design of integrated knowledge acquisition systems.

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