

## TRENDS IN STOCHASTIC COMPUTING

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The original development of the stochastic computer took place independently at Illinois and STL in 1965. Some 11 years later there is no dramatic progress to report and the 1969 technical survey<sup>1</sup> is still adequate in most respects. However, the problem discussed therein, of the difficulty of simulating large systems (e.g. distillation columns) in real time, still remains. Meanwhile there has been steady progress in stochastic computing on several fronts:

Stochastic Computer Implementation The Illinois/STL groups built only small prototype machines and most studies since have been by simulation. This in itself is a hindrance to progress and we are fortunate that one group in Britain, at Aberdeen, have been funded to build a stochastic hybrid on a reasonable scale (34 integrators linked to a PDP8 with realtime IO), and have investigated its performance in a range of applications including matrix inversion, linear programming and process simulation<sup>2,3</sup>. A large-scale hybrid has been proposed by a commercial company in Canada who have developed a variable step-size stochastic integrator<sup>4</sup> allowing sine-wave generation at 300 Hz with 1% accuracy<sup>5</sup>. Other implementations have been on a smaller scale but widespread in USA, Japan, France, Spain, Portugal, Austria, etc.

Applications of Stochastic Computing Apart from the general-purpose hybrids above, there have been interesting applications of stochastic computing techniques to specific problems. The original 'Enhancetron' biological transient averager<sup>1</sup> was based on stochastic analog/digital (AD) conversion. Commercial MOS microcircuits using similar techniques for low-cost DA/AD conversion have now been on sale for some years based on work in France<sup>6</sup>. In Vienna a family of measuring instruments, waveform analysers, correlators, etc., has been developed and offered commercially<sup>7</sup>. All of these devices use pseudo-random, rather than true random, sources, but their design and operation is based on stochastic computation techniques allowing low-cost, high packing-density, analog computation with digital lsi microcircuits.

Developments of the Stochastic Computer The pseudo-stochastic instruments mentioned have been one byproduct of work on stochastic computing, as has the 'phase' computer<sup>8</sup>. In particular, at Illinois the work first moved on to concentrate on ultra-reliability through distributed stochastic computation (the 'bundle' machine) and then onto faster, but still simple, pseudo-stochastic processing (the 'burst' machine) which have been combined to allow high-speed, low-cost, ultra-reliable systems to be implemented, including a digital radio receiver<sup>9,10</sup>. Another byproduct has been the study of stochastic computation using the natural random processes in fluidic elements in Germany<sup>11,12</sup>. Nonlinear stochastic representations have also been developed

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which allow computations to take place with numbers in an infinite range and have a logarithmic (slide-rule) error characteristic<sup>13</sup>.

Where Stochastic Computation is Essential The first developments of stochastic computation saw it as a basis for low-cost, high-density analog computation with digital lsi microcircuits. A fascinating feature of some later studies has been the demonstration that there are computations where randomness is essential<sup>14,15,16</sup>. The first result was that adaptive threshold logic elements with discrete weights failed to adapt unless weight changes were random<sup>1</sup>. This was generalized to show that there was a class of control problems soluble by a 2-state stochastic automaton but insoluble by any finite-state deterministic automaton<sup>17</sup>. Independently it was also shown that recursive automata could solve these problems<sup>18</sup> and hence that there was an equivalence in power between simple finite-state stochastic algorithms and complex, potentially infinite-state deterministic algorithms. These and similar results have implications both for artificial intelligence and biological systems modelling.

Thus, the current trends in stochastic computing are diverse, from commercial microcircuits and instruments, through process simulation, to learning automata and biological modelling. The surprising possibility of utilizing noise has been accepted<sup>19</sup>, stochastic computing has reached the textbooks, and it seems that one more computational technique has been assimilated into the armoury of systems engineers.

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